Math 266B: Homework 6-(1) Due Feb. 27th

1. Show that the solution $u(x,y)$ to $u_y - xuu_x = 0$ with $u(x,0) = x$ is defined implicitly by $x = ue^{-yu}$. Show further that the characteristic curves for this problem are all tangent to the curve $y = 1/(ex)$. What should be the value of u in the plane between the two components of the graph of $y = (ex)^{-1}$?

2. Find the characteristics for the initial value problem

$$w_y = \frac{1}{2}((w_x)^2 + x^2)$$

with the initial condition $w(x,0) = x$. The solution will not be defined for $|y| > \pi/2$. Explain that from the behavior of the characteristics.

3. Consider the initial value problem

$$\begin{cases} w_x + w_x w_y = 1, \\ w(x,0) = f(x). \end{cases}$$

When is this problem noncharacteristic? When it is noncharacteristic, show that the solution near $y = 0$ is given by

$$w(x,y) = f(t) - y + \frac{2y}{f'(t)},$$

where $t(x,y)$ is defined by the equation

$$y = (f'(t))^2(x - t).$$

Explain why the solvability of that equation for $t(x,y)$ with $t(x,0) = x$ is equivalent to the noncharacteristic condition at $(x,0)$.