Fall 2009 266A: Homework 6. Due Nov. 13th

1. B and N p 208 Exercise 1.

2. Let $x, y \in I\!R^n$ and consider the Hamiltonian system

$$x' = y, \quad y' = -\nabla U(x),$$

where $U : I\!R^n \to I\!R$ is C^2, with $\nabla U(a) = 0$. Show the following:

(a) If a is a strict local minimum of U, then a is stable;

(b) If a is a local maximum of U and if D^2U is an invertible matrix, then $(a, 0)$ is unstable.

(Hint: you may consider another Lyapunov function $V(x, y) = \Sigma_{i=1}^n x_i y_i$.

3. Consider the equation of motion for an undamped simple pendulum, $my'' = -\sin y, |y| \leq 2\pi$.

(a) Show that the equation can be written as a Hamiltonian system.

(b) Determine the stability of the stationary points of the system.

(c) Does the linearization theorems we discussed in chapter 4 apply for the system, near $(0, 0)$?

4. B and N p215, Exercise 2.

5. Consider the equation

$$x'' + x' + 4x^3 - 6x^2 + 2x = 0.$$

For the corresponding system, fInd the critical points, determine their stability properties and their basin of attraction.