

Fall 2009 266A: Homework 4. Due Oct. 28th

1. B and N p.161, Exercise 1. Assume that $F(t, y)$ is defined in $D = [0, \infty) \times \mathbb{R}^n$, and F_y is uniformly continuous in D .

2. B and N p. 164 Exercise 2.

3. Consider the equation

$$y' = A(t)y + g(t, y),$$

where $A(t)$ is continuous and periodic of period p and g is continuous, satisfy a local Lipschitz condition, and

$$\lim_{\|y\| \rightarrow 0} \sup_{t > t_0} \frac{\|g(t, y)\|}{\|y\|} = 0.$$

Let R be the matrix given in Floquet Theorem. Show that $y = 0$ is stable if all the negative eigenvalues of R have negative real part and is unstable if at least one eigenvalue of R has positive real part.

4. Suppose that $f(y) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is continuously differentiable, and $f(0) = 0$. If all eigenvalues of $f'(0) = 0$ have positive real parts, show that the zero solution to $y' = f(y)$ is not stable (Be careful to cover all cases) .

5. Consider the equation

$$x' = y - \mu x(x^2 + y^2), y' = -x - \mu y(x^2 + y^2).$$

The point $(0,0)$ is a critical point. Show that $(0,0)$ is stable with the linearized system around $(0, 0)$. Next show that for $\mu < 0$ the point $(0, 0)$ is unstable.