Fall 2009 266A: Homework 3. Due Wed. Oct. 21st

1. Suppose $A(t)$ is a $n \times n$ matrix with continuous coefficients. Further suppose that the matrix $B(t) = \int_{t_0}^{t} A(s)ds$ commutes with $A(t)$. Show that, by performing Picard iteration starting with $\phi_0(t) \equiv y_0$, the solution of

$$y' = A(t)y, \quad y(t_0) = y_0$$

is given by $y(t) = exp^{\int_{t_0}^{t} A(s)ds} y_0$. Note that this argument avoids differentiation of infinite series. In particular when A is a constant matrix, $B(t)$ commutes with A and thus the assertion is true.

2. Consider $y' = A(t)y$ with

$$A(t) = \begin{pmatrix} t & 1 \\ 0 & 0 \end{pmatrix}$$

Show that $A(t)$ does not commute with $B(t)$, and find the resolvent $R(t, 0)$.

3.

(a) Compute the resolvant $R(t, 0)$ for the ODE

$$x' = \cos(t)x - \sin(t)y, \quad y' = \sin(t)x + \cos(t)y$$

Hint: Find an equation for the complex function $z = x + iy$.

(b) Determine $P(t)$ and R in Floquet theorem.

4. Consider the equation for the mathematical pendulum

$$x'' + \sin(x) = 0, x(0) = \epsilon, x'(0) = 0,$$

where ϵ is supposed to be small. Show, with Taylor expansion, that the solution can be written in the form

$$x(t) = \epsilon x_1(t) + \epsilon^2 x_2(t) + O(\epsilon^3).$$

Compute $x_1(t)$ and $x_2(t)$.

5. Draw the phase space for the competing species system

$$x' = x(2 - x - y), y' = y(3 - 2x - y)$$

in the first quadrant. How likely is it that both species survive?