Fall 2009 266A: Homework 1. Due Oct. 7th

1. Brauer and Nohel p110, Ex. 2

2. B and N p. 118, Ex. 12

3. Consider $y' = |y|^{-3/4}y + f(t)$ with $f(t) = t\sin(\pi/t)$ when $t \neq 0$ and $f(0) = 0$. Show that the Forward Euler Scheme with initial data $y(0) = 0$ and $k = (n+1/2)^{-1}$ has at least two different subsequential limits as $n \to \infty$ in any time interval containing $t = 0$.

Hint: show that for k : *even*, $y_k(t) \geq t^{3/2}/6$ for $3k \leq t \leq 1/5000$, and for k : *odd*, $y_k(t) \leq -t^{3/2}/6$ for $3k \leq t \leq 1/5000$.

4. Consider the Fitzhugh-Nagumo equation

$$y_1' = f_1(y_1, y_2) = g(x_1) - x_2, y_2' = f_2(y_1, y_2) = \sigma y_1 - \gamma y_2, \qquad (1)$$

where σ and γ are positive constants and the function g is given by $g(x) = -x(x-1/2)(x-1)$.

(a) In the x_1-x_2 plane draw the graph of the curves $f_1(x_1, x_2) = 0$ and $f_2(x_1, x_2) = 0$.

(b) Consider the rectangles whose sides are parallel to the x_1 and x_2-axis, with two opposite corners located on $f_2(x_1, x_2) = 0$. Show that if the rectangle is taken sufficiently large, a solution which start inside the rectangle stays inside the rectangle forever. Deduce from this that the solution $(y_1(t), y_2(t))$ for any initial conditions at $t = 0$ have a unique solution for all time $t > 0$.

5. B and N, p. 140, Ex. 6. Show one example where uniqueness fails without the Osgood condition.