

**Exercises from week 5:**

1. Show that there is  $u \in C^\alpha(U)$  (for any  $\alpha \in (0, 1)$ , or for some  $\alpha \in (0, 1)$  of your choice) such that  $Du \notin L^1(U)$ .

2. Let  $1 < p < \infty$  and let  $f \in L^q(U)$ , where  $1/p + 1/q = 1$ . Show that there is a minimizer in  $W_0^{1,p}(U)$  of the following energy:

$$E(u) = \int (|Du|^p + fu) dx.$$

3. Show that for any  $u \in BV(U)$ , there is a sequence  $u_k \in C^\infty(U)$  such that

(a)  $u_k \rightarrow u$  in  $L^1(U)$ ;

(b)  $\int |Du_k| dx \rightarrow \int |Du| dx$ .