

Exercises from week 4:

1. Find a function $u \in \mathbb{R}^2 \rightarrow \mathbb{R}$ which is bounded, compactly supported, and is not equal to a continuous function almost everywhere, such that u has a weak derivative.

2-3. Evans chapter 5 problem 7,9.

4. Prove the Extension Theorem for $p = \infty$.

5. Let U be a bounded domain in \mathbb{R}^d with $\partial U \in C^1$, and let $1 \leq p < \infty$. Show that if $u \in H_0^1(U)$ then for any $\phi \in C_c^\infty(\mathbb{R}^d)$ we have

$$\int_U u(x) D\phi(x) dx = - \int_U Du(x) \phi(x) dx.$$

Does the other direction (“only if”) also hold?