Math 251A Fall 2026: Homework 3, part I

1. Complete the proof of Lemma 2 in Section 3.3 of Evans, by showing that u is Lipschitz continuous at all times.

2-3. Evans second edition, problem 13, 16.

4. Let $u \in C^2(\mathbb{R}^d \times [0,T])$ solve

$$(P) \qquad \begin{cases} u_t + H(Du) = 0 & \text{in } \mathbb{R}^d \times (0,T], \\ u(\cdot,0) = g. \end{cases}$$

(a) Show that $p := Du$ solves $p_t + V \cdot Dp = 0$, where $V := DH(Du) = 0$.

(b) Show that the optimal path starting at y is given by $x = y + t(DH(Dg(y)))$.

(c) Let $d = 1$. If in addition g' is bounded, show that $\Phi_t(y) := y + tH'(g'(y)))$ is injective and monotone increasing for each $t > 0$. Conclude that u is given by

$$u(x,t) := g(y) + tL(H'(g'(y))), \quad y = \Phi_t^{-1}(x).$$