Spring 2017 MATH 134: Homework 2, due April 21st

Note: At least one of the homework problems, including the suggested exercises, will appear on each exam.

1. Consider the equation $\dot{x} = rx + x^3$ where $r > 0$ is fixed. Show that $x(t) \to +\infty$ or $x(t) \to -\infty$ in finite time, starting from any initial condition $x(0) \neq 0$. Hint: you just have to show the time is finite, so it might simplify things to use the bound ($rx + x^3 > x^3$ for $x > 0$).

2. For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

(a) $\dot{x} = -\cos x$;

(b) $\dot{x} = -r + 2x - 3x^2$ for $r = 1, 1/4, 1/8, -1$.

3. For each of the following exercises, sketch all qualitatively different phase portraits that occur as r is varied. (Phase portraits are *qualitatively different* if there are different numbers/types of fixed points.) Show that a saddle-node bifurcation occurs at a critical value of r, to be determined. Finally, sketch the bifurcation diagram of fixed points x_* versus r. (For (b)-(c) You will NOT be able to solve for as a function of r. Instead, just do your best to sketch the bifurcation diagrams roughly.)

(a) $\dot{x} = r - x^4$;

(b) $\dot{x} = r - x + \ln(1 + x)$;

(c) $\dot{x} = r + x - \frac{2x}{1+x}$

4. For each of the following equations, sketch all qualitatively different phase portraits that occur as r is varied. Show that a transcritical bifurcation occurs at a critical value of r, to be determined. Finally, sketch the bifurcation diagram of fixed points x_* versus r. (You should be able to sketch the bifurcation diagrams without a computer.)

(a) $\dot{x} = rx - \ln(1 + x)$, suppose $x > -1$;

(b) $\dot{x} = x(r - e^x)$.

5. Consider the system $\dot{x} = rx - x^3 + x^5$.

(a) Find algebraic expressions for all the fixed points as r varies.

(b) Sketch the bifurcation diagram, be sure to indicate all the fixed points and their stability.

(c) Find all r where the nonzero fixed points are born in a saddle-node bifurcation.

Other suggested exercises not to be turned in :
Strogatz 2.7.7, 3.1.5, 3.4.10, 3.4.11.