Math 134 Spring 2017 Lecture 2: Homework 1, due 4/14

Note: At least one of the homework problems, including the suggested exercises, will appear on each exam.

1. Let $y(t) : \mathbb{R} \to \mathbb{R}$ solve the third order ODE

$$y''' = y^2 + t^2.$$

(a) Define x and $f(x)$ so that the equation may be rewritten in the form $\dot{x} = f(x)$.

(b) What is the dimension of the system $\dot{x} = f(x)$?

2. Let $x(t) : \mathbb{R} \to \mathbb{R}$ solve the ODE

$$\dot{x} = x - \cos x.$$

Let x_0 be the fixed point of the ODE that lies between zero and $\pi/2$ (explain why it exists, by drawing the graph of x and $\cos x$). Determine the stability of x_0.

3. Consider the ODE

$$\dot{x} = 3\cos x.$$

(a) Find all fixed points of the ODE.

(b) At which points x does the flow have the greatest velocity to the left?

(c) Find the flow's acceleration x'' as a function of x.

(d) At which points do the flow have maximum positive acceleration?

4. In each of the following collection of fixed points, give an example of an equation $\dot{x} = f(x)$ that fits the description;

(a) $x = 0$: unstable, $x = 1$: stable and $x = 2$: semistable;

(b) $x = -1, 1$: stable and $x = 0$: unstable.

5. There are two ways to solve the logistic equation $N' = rN(1 - N/K)$ analytically for an arbitrary initial condition N_0.

(a) Separate variables and integrate, using partial fractions.

(b) Make the change of variables $x = 1/N$. Then solve the resulting differential equation for x.

Solve the equation in both ways (Assume $K, r > 0$).

6. A particle travels on the half line $x \geq 0$ with a velocity given by $\dot{x} = ax^{1/b}$, where $a, b \in \mathbb{R}$.

(a) Find all values of a and b so that the origin $x = 0$ is a stable fixed point.

(b) Suppose a and b are chosen such that $x = 0$ is stable. Can the particle ever reach the origin in a finite time? Specifically, how long does it take for the particle to travel from $x = 1$ to$x = 0$, as a function of a and b?(Hint: since distance = (rate)(time), the time it takes to move a distance dx is given by $dt = dx/v(x)$, where $v(x)$ is the velocity at x.)

(c) For the values of a and b so that the particle reaches the origin in finite time, is $ax^{1/b}$ continuously differentiable on $[0, \infty)$?

7. Suppose $x(t)$ solves $\dot{x} = f(x)$ with $f(0) = 0$ and $|f(x)| \leq A|x|$. Show that if $x(0)$ is not equal to zero then $x(t)$ can never reach the stationary solution $x = 0$ in finite time.

Other suggested exercises that need not be turned in:
Strogatz 2.2.9, 2.3.4, 2.4.2, 2.4.4, 2.4.8, 2.5.4, 2.5.6, 2.6.2.