1. Let \( y(t) : \mathbb{R} \to \mathbb{R} \) solve the third order ODE
   \[
y''' = y^2 + t^2.
   \]
   (a) Define \( x \) and \( f(x) \) so that the equation may be rewritten in the form \( \dot{x} = f(x) \).
   (b) What is the dimension of the system \( \dot{x} = f(x) \)?

2. Let \( x(t) : \mathbb{R} \to \mathbb{R} \) solve the ODE
   \[
   \dot{x} = x - \cos x.
   \]
   Let \( x_0 \) be the fixed point of the ODE that lies between zero and \( \pi/2 \) (explain why it exists, by drawing the graph of \( x \) and \( \cos x \)). Determine the stability of \( x_0 \).

3. Consider the ODE
   \[
   \dot{x} = 3 \cos x.
   \]
   (a) Find all fixed points of the ODE.
   (b) At which points \( x \) does the flow have the greatest velocity to the left?
   (c) Find the flow’s acceleration \( x'' \) as a function of \( x \).
   (d) At which points do the flow have maximum positive acceleration?

4. In each of the following collection of fixed points, give an example of an equation \( \dot{x} = f(x) \) that fits the description;
   (a) \( x = 0 \): unstable, \( x = 1 \): stable and \( x = 2 \): semistable;
   (b) \( x = -1, 1 \): stable and \( x = 0 \): unstable.
5. There are two ways to solve the logistic equation \( N' = rN(1 - N/K) \) analytically for an arbitrary initial condition \( N_0 \).

(a) Separate variables and integrate, using partial fractions.

(b) Make the change of variables \( x = 1/N \). Then solve the resulting differential equation for \( x \).

Solve the equation in both ways (Assume \( K, r > 0 \)).

6. A particle travels on the half line \( x \geq 0 \) with a velocity given by \( \dot{x} = ax^{1/b} \), where \( a, b \in \mathbb{R} \).

(a) Find all values of \( a \) and \( b \) so that the origin \( x = 0 \) is a stable fixed point.

(b) Suppose \( a \) and \( b \) are chosen such that \( x = 0 \) is stable. Can the particle ever reach the origin in a finite time? Specifically, how long does it take for the particle to travel from \( x = 1 \) to \( x = 0 \), as a function of \( a \) and \( b \)? (Hint: since distance = (rate)(time), the time it takes to move a distance \( dx \) is given by \( dt = dx/v(x) \), where \( v(x) \) is the velocity at \( x \).)

(c) For the values of \( a \) and \( b \) so that the particle reaches the origin in finite time, is \( ax^{1/b} \) continuously differentiable on \([0, \infty)\)?

7. Suppose \( x(t) \) solves \( \dot{x} = f(x) \) with \( f(0) = 0 \) and \( |f(x)| \leq A|x| \). Show that if \( x(0) \) is not equal to zero then \( x(t) \) can never reach the stationary solution \( x = 0 \) in finite time.

Other suggested exercises that need not be turned in:
Strogatz 2.2.9, 2.3.4, 2.4.2, 2.4.4, 2.4.8, 2.5.4, 2.5.6, 2.6.2.