1. [Gronwall’s inequality exercise] Let us consider the system
\[
\dot{x} = -x(1 + x^2) + y, \quad \dot{y} = -x - y(1 + x^2).
\]
Show that \((x^2 + y^2)(t) \leq e^{-2t}(x^2 + y^2)(0)\). It follows that any trajectory in the system converges to the origin exponentially fast.

Other suggested exercises not to be turned in: Strogatz 8.2.9 (a-c), 8.2.12