Spring 2017 MATH 134: Homework 6, due May 26th

Note: At least one of the homework problems, including the suggested exercises, will appear on each exam.

1. Let us consider the system

$$\dot{x} = y - y^3, \quad \dot{y} = -x - y^2.$$

(a) Plot the nullclines $\dot{x_1} = 0$ and $\dot{y} = 0$.

(b) Find the signs of $\dot{x_1}$ and $\dot{y}$ on the regions of the plane separated by the nullclines.

(c) Find the eigenvalue and eigenvectors (for real eigenvalues only) at all the fixed points.

(d) Consider the unstable manifold[1] of $(-1, -1)$. Prove that this submanifold eventually reaches the x-axis (for this you must show $\dot{y}$ has the right sign).

(e) With the above argument, and using the reversibility of the system, show there are two heteroclinic orbits connecting $(-1, 1)$ and $(-1, -1)$.

2. Strogatz 6.8.1.

3. Use index theory to show that the system

$$\dot{x} = x(y - 1), \quad \dot{y} = y(4 - x - y^2)$$

has no closed orbits. (Hint: you may use that the unstable manifold corresponding to the saddle at $(0, 0)$ converges to the stable spiral. This is not obvious, but you need this fact to solve the problem.)

4. Strogatz 6.8.8.

5. Strogatz 7.2.7.

Other suggested exercises not to be turned in: Strogatz 6.8.9, 7.2.5, 7.2.9.

[1] recall this is the set of points such that if you go backward in time starting at that point then as $t \to -\infty$ you approach $(-1, -1)$.