Spring 2017 MATH 134: Homework 4, due May 5th

Note: At least one of the homework problems, including the suggested exercises, will appear on each exam.

1. Strogatz 4.5.1

2. Consider the system $\dot{x} = y, \dot{y} = x$.

(a) Sketch the vector field.

(b) Show that the trajectories of the system are hyperbolas of the form $x^2 - y^2 = C$. (Hint: show that the governing equations imply $x\dot{x} - y\dot{y} = 0$ and then integrate both sides.)

(c) The origin $x_* = 0$ is a saddle point; find equations for its stable and unstable manifolds. (The **stable manifold** is the set of initial conditions x(0) so that $\lim_{t\to+\infty} x(t) = x_*$. The **unstable manifold** is the set of initial conditions $x(0)$ so that $\lim_{t\to-\infty} x(t) = x_*$.)

(d) The system can be decoupled and solved as follows. Introduce new variables u and v, where $u = x + y, v = x - y$. Then rewrite the system in terms of u and v. Solve for $u(t)$ and $v(t)$ starting from an arbitrary initial condition (u_0, v_0).

(e) Finally, use the answer to part (d), write the general solution for $x(t)$ and $y(t)$ starting from an initial condition (x_0, y_0).

3. Strogatz 5.1.10 (you do not need to use the definition given in the exercise, you can describe the stability types using the informal description given in the class).

Other suggested exercises not to be turned in : Strogatz 4.5.3, 5.1.7, 5.2.2.