**Math 131BH Spring 2018: Homework 9, Due 6/8**

1. Suppose $f : [0,1] \to \mathbb{R}$ is bounded and $f \in \mathcal{R}(x)$ in $(\delta, 1]$ for any $\delta > 0$. Show that $f \in \mathcal{R}(x)$ in $[0,1]$. This exercise is to verify that $g(x)$ given in the proof of Theorem 8.14 in Rudin is integrable.

2. Let $f \in \mathcal{R}(x)$ in $[-\pi, \pi]$ with $\int_{-\pi}^{\pi} f(x)dx = 0$ and let $F(x) := \int_{-\pi}^{x} f(t)dt$. Show that

(a) $s_N(F)$, the $N-th$ Fourier series of $F$, uniformly converges to $F$ on $[-\pi, \pi]$ as $N \to \infty$.

(b) $\hat{F}(n) = \frac{\hat{f}(n)}{in}$ for $n \neq 0$. Can $\hat{F}(0)$ be written in terms of $\hat{f}(n)'s$?

(c) Is $(a)$ true when $\int_{-\pi}^{\pi} f(x)dx \neq 0$? Explain.

3-5. Rudin Chapter 9, Problems 6,7,9.