Math 131BH Spring 2018: Homework 8, due in class 6/1

1. A finite collection of functions $\{\phi_i\}_{i=1}^{\infty} \subset L^2([-\pi,\pi])$ is called *linearly independent* if the equation

$$\int_{-\pi}^{\pi} |\Sigma_{k=0}^{m} c_k \varphi_k|^2 dx = 0$$

implies $c_0 = c_1 = ... = c_m = 0$. An infinite collection is called linearly independent if every finite subset is linearly independent. Prove that every orthonormal system in $L^2([-\pi,\pi])$ is linearly independent.

2. Suppose that $f \in C^1([-\pi,\pi])$, $f(-\pi) = f(\pi)$, and that $\int_{-\pi}^{\pi} f(t)dt = 0$. Prove that $\|f'\|_{L^2} \geq \|f\|_{L^2}$, with equality if and only if $f(x) = a\cos x + b\sin x$.

3-7. Rudin 12, 14, 15,16,19.