Math 131BH Spring 2018: Homework 5, Due 5/9

1-2. Rudin problems 16,18

3. Let $f_n : [0,1] \to \mathbb{R}$ be continuous with the property

$$|f_n(x)| \leq 1 + \frac{n}{1+n^2x^2},$$

and define $F_n : [0,1] \to \mathbb{R}$ by

$$F_n(x) := \int_0^x f_n(t)dt.$$

Show that the sequence (F_n) has a subsequence that converges pointwise on $[0,1]$.

4. Let $g \in C^1([-2,2])$, $g = 0$ outside $[-1,1]$, and $\int_{-1}^1 g(x)dx = 1$. Show that for any sequence $f_n \in C([-1,1])$ the sequence $(\tilde{f}_n)$ given by

$$\tilde{f}_n(x) := \int_{-1}^1 g(x-y)f_n(y)dy$$

is equicontinuous on $[-1,1]$, and satisfies $\int_{-2}^2 \tilde{f}_n(x)dx = \int_{-1}^1 f_n(x)dx$.

5. [Optional] Let (f_n) be a sequence in $C([-1,1])$ such that

(a) f_n is uniformly bounded;

(b) f_n converges pointwise to a continuous function f on $[-1,1]$ as $n \to \infty$.

Show that $\int_{-1}^1 f_n(x)dx$ converges to $\int_{-1}^1 f(x)dx$. Note that this is not true if we omit (a).