Math 131BH Spring 2018: Homework 4, Due 5/2

1-3. Rudin p165, problems 3,5,8.

4. Let (f_n) be a sequence of function from $[0,1] \to \mathbb{R}$ such that f_n converges pointwise to a continuous function f on $[0,1]$ as $n \to \infty$. Is it always true that $\int_0^1 |f_n(x) - f(x)|^2 dx \to 0$ as $n \to \infty$? Explain.

5. For each $n \geq 1$ let $f_n : [-1,1] \to \mathbb{R}$ be a continuous, nonnegative function and assume that

(a) $\int_{-1}^1 f_n(y)dy = 1$;

(b) For each $c > 0$, (f_n) uniformly converges to 0 on $[-1,-c] \cup [c,1]$.

Show that for every continuous function $g : [-1,1] \to \mathbb{R}$, we have

$$\lim_{n\to\infty} \int_{-1}^1 f_n(y)g(y)dy = g(0).$$

6. Let (f_n) be as given above, and in addition suppose f_n is supported on $[-1/n, 1/n]$, $f_n(x) = f_n(-x)$ and is in $C^1([-1,1])$. Consider a continuous function in $g : [-2,2] \to \mathbb{R}$. Consider

$$h_n(x) := \int_{-2}^2 f_n(y-x)g(y)dy,$$

where we extend f_n to be zero when y is outside of $[-1,1]$. Show that

(a) $h_n \in C^1([-1,1])$;

(b) h_n uniformly converges to g on $[-1,1]$ as $n \to \infty$.