Math 131BH Spring 2018: Homework 2, due in class 4/18

1-3. Rudin p114, Problems 15,18,25.

4. Let f and g be n-th differentiable in (a,b), and suppose that for some $c \in (a,b)$ we have $f(c) = f'(c) = ... = f^{(n-1)}(c) = 0$, and $g(c) = g'(c) = ... = g^{(n-1)}(c) = 0$, but that $g^{(n)}(x)$ is never zero in (a,b). Show that

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f^{(n)}(c)}{g^{(n)}(c)},$$

if $g(x) \neq 0$ for $x \neq c$.

5-6. Rudin p138, Problems 3,5.

7. Let $f, \alpha : [a,b] \to \mathbb{R}$ be two monotone increasing functions, and suppose $f \in \mathcal{R}(\alpha)$ on $[a,b]$. Show that $\alpha \in \mathcal{R}(f)$ on $[a,b]$ and

$$\int_a^b f d\alpha + \int_a^b \alpha df = f(b)\alpha(b) - f(a)\alpha(a).$$