Math 131BH Spring 2018: Homework 1, due in class 4/11

1. Let $f : [0,1] \to \mathbb{R}$ be a one-to-one function which is differentiable in $[0,1]$.

(a) Show that f is monotone in $[0,1]$.

(a) Explain why f^{-1} is continuous.

(b) In addition suppose that $f'(x) \neq 0$ for $x \in I := f([0,1])$. Show that the inverse function f^{-1} is differentiable at x and

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))} \text{ for } x \in I.$$

2. This exercise is to review the concept of complete metric space from last quarter, and also for later discussions in class on various function spaces and the appropriate metrics for them. Let us define the function space

$$X = C^1([0,1]) := \{f : [0,1] \to \mathbb{R}, f \text{ is continuously differentiable in } [0,1]\},$$

and for $f, g \in X$ define $d(f,g) := \sup_{x\in[0,1]} |f(x) - g(x)|$.

(a) Show that d is a metric in X.

(b) Show that the metric space (X, d) is not complete.

3. Let $f : [a,b] \to \mathbb{R}$ be continuous, and suppose that f is differentiable in (a,b). Show that for any $c \in (a,b)$ that is not a point of maximum or minimum for f' there exist $x_1, x_2 \in (a,b)$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

4-8. Rudin Chapter 5 problem 6, 8, 9, 26, 27.