1-5. Rudin Chapter 5, problems 11,15,16,19,22.

6. Let \( \alpha(x) := 1 \) for \( x \geq 0 \) and \( \alpha(x) := 0 \) for \( x < 0 \), and let \( f = \alpha \). Show that \( f \notin \mathcal{R}(\alpha) \).

7. Let \( f, \alpha : [a, b] \rightarrow \mathbb{R} \) be monotone increasing functions. \( f \in \mathcal{R}(\alpha) \) on \( [a, b] \).
Then show that \( \alpha \in \mathcal{R}(f) \) on \( [a, b] \) and
\[
\int_a^b f \, d\alpha + \int_a^b \alpha \, df = f(b)\alpha(b) - f(a)\alpha(a).
\]

8. Suppose \( f \) is monotone and is differentiable for “almost every” \( x \) in \([0, 1]\)
in the following sense: For any \( \varepsilon > 0 \), there exists a union of disjoint open
intervals \( A = \bigcup_{n=1}^{N} I_n \) in \([0, 1]\) such that
(a) \( \Sigma_{n=1}^{N} |I_n| \geq 1 - \varepsilon \) and
(b) \( f \) is differentiable in \( A \).

Is it always true that \( f(1) - f(0) = \int_0^1 f'(x) \, dx \)?