Winter 2012 MATH 131BH Midterm

Please write clearly, and show your reasoning with mathematical rigor.

Name: Student ID:

Problem	Score
1	
2	
3	
4	
Total	

1. (12pt)

(a) Let $\alpha : [0,1] \to I\!R$ be a monotone increasing function and suppose $f, g \in I\!R(\alpha)$. Show that $f + g \in I\!R(\alpha)$.

(b) Find $\alpha : [0,1] \to I\!R$ such that

$$\int_0^1 f(x)d\alpha = \int_{1/2}^1 f(x)dx + f(1/2)$$

for every continuous $f : [0,1] \to I\!R$. (No proof is necessary)

2. (12pt) (True or False) If true, you do not need to prove it. If false, please provide a counterexample: you do not need to prove that it is a counterexample.

(a) Let $f, \alpha : [a, b] \to I\!R$. If $f \in \mathcal{R}(\alpha)$ then $\alpha \in \mathcal{R}(f)$.

(b) Let $\alpha : [a, b] \to I\!R$ be monotone increasing. If $f_n \in \mathcal{R}(\alpha)$ and $f_n \to f$ pointwise then $f \in \mathcal{R}(\alpha)$.

(c) Suppose $f : (0, 1) \to I\!R^n$ is differentiable. Then for any vector $\eta \in I\!R^n$ the function $g(x) =< f(x), \eta >: (0, 1) \to I\!R$ is differentiable.

(d) If $f : (-1, 1) \to I\!R$ is three times differentiable and satisfies $f = f' = f'' = 0$ at $x = 0$, then $f(x) \leq C|x|^3$ for some constant C on $(-1, 1)$.

3. (12pt) Suppose that $f^2 \in \mathcal{R}(\alpha)$ and $\int_0^1 f^2(x)d\alpha = 0$. Show that then $f \in \mathcal{R}(\alpha)$ and $\int_0^1 f(x)d\alpha = 0$. (Hint: use the fact that $|f| \leq (f/\epsilon)^2 + (\epsilon)^2$ for any $\epsilon > 0$ (why is this true?))

4. (14pt) Let $f_n : [0,1] \to I\!R$.

(a) Show that if f_n's are continuous and f_n uniform converges to f then f is continuous on $[0,1]$.

(b) If $f_n \to f$ pointwise on $[0,1]$, $|f_n| \leq 1$, and f is continuous on $[0,1]$, can you say that $f_n \to f$ uniformly on $[0,1]$? If it is false then provide a counterexample.