Math 131BH Homework 3: due Jan.30th

1-4. Rudin p166, Ex. 5,6,9 (If the converse is false provide a counterexample), 10.

5. Let X be a metric space with distance d_X. In class we have defined the metric space $\mathcal{C}(X)$ with the distance associated with the sup-norm

$$d(f,g) = \sup_{x \in X} |f(x) - g(x)|$$

and showed that it is a complete metric space. Show that $\mathcal{C}(X)$ is connected but not compact.

6. Consider the subset of $\mathcal{C}([0,1])$ defined as follows:

$$X = \{f : [0,1] \to \mathbb{R}, f(0) = 0, |f(x) - f(y)| \leq |x - y|\}.$$

Prove that X is compact.