Math 131BH Winter 2012: Homework 1, Due 1/16

0. (Review question) For any subset A of $\mathbb{R}$ which is nonempty and bounded above, show that $\sup A = -\inf(-A)$. As a consequence show that, for any interval $[a,b]$ and a bounded function $f : [a,b] \to \mathbb{R}$, we have

$$\sup_{[a,b]} f - \inf_{[a,b]} f \leq 2 \sup_{[a,b]} |f|.$$

1-3. Rudin p 138, Ex. 3,6,8.

4. Let $f, \alpha : [a,b] \to \mathbb{R}$ be monotone increasing functions. $f \in \mathcal{R}(\alpha)$ on $[a,b]$. Then show that $\alpha \in \mathcal{R}(f)$ on $[a,b]$ and

$$\int_a^b f d\alpha + \int_a^b \alpha df = f(b)\alpha(b) - f(a)\alpha(a).$$

5. (option to postpone until homework 2) Compute

$$\int_0^1 x d\alpha,$$

where α is the Cantor function constructed in class.

6. Suppose f is monotone and is differentiable for "almost every" x in $[0,1]$ in the following sense: For any $\varepsilon > 0$, there exists a union of disjoint open intervals $A = \cup_{n=1}^N I_n$ in $[0,1]$ such that

(a) $\Sigma_{n=1}^N |I_n| \geq 1 - \varepsilon$ and

(b) f is differentiable in A.

Is it always true that $f(1) - f(0) = \int_0^1 f'(x)dx$?