Math 131AH Winter 2018: Homework 4, Due 2/7

1. Let E be a non-empty subset of $\mathbb{R}$ that is both open and closed. Show that $E = \mathbb{R}$.

2. Show that any open subset of $\mathbb{R}$ is a countable or finite union of disjoint open intervals.

3. For the space of polynomials

$$X := \{P(x) = \Sigma_{k=1}^{n} a_k x^k, n \in \mathbb{N} \text{ and } a_k \in \mathbb{R}\},$$

let us define

$$d(P, Q) := (\Sigma_{k=1}^{m} |a_k - b_k|^2)^{1/2}$$

for two polynomials $P(x) = \Sigma_{k=1}^{n} a_k x^k$ and $Q(x) = \Sigma_{k=1}^{m} b_k x^k$ in X, with $m \geq n$: we define $a_k = 0$ if $k \geq n$.

(a) Show that d is a metric on X.

(b) Let 0 be the constant zero polynomial. Show that the set $B := \{P(x) \in X : d(P, 0) \leq 1\}$ is closed and bounded but not compact.

4-6. Rudin p. 43 (Chapter 2.) Exercises 9,12,16.