Math 131AH Winter 2018: Homework 3, Due 1/31

1. Let E be a nonempty set. Show that E is infinite if and only if E has the same cardinality with at least one of its proper subset.

2.

(a) Let X be countable, and let Y be an infinite subset of X. Show that Y is countable. You can follow the proof in the book if you would like to, the point is to make you understand the proof.

(b) Using (a), show that if $\{X_n\}_{n=1}^{\infty}$ is a family of countable sets, then $\cup_{n \in N} X_n$ is countable. Explain where we need (a).

3. Let us denote $(0,1) := \{x : x \in \mathbb{R}, 0 < x < 1\}$. For $x \in (0,1)$, let $f(x) = (a_1, a_2, a_3, ...) \subset \{0,1,...,9\}^{\mathbb{N}}$, where the $a_k \in \{0,1,...,9\}$ are chosen uniquely by the decimal expansion constructed in class. Recall that in particular we showed that $r_n := 0.a_1a_2..a_n$ satisfies $r_n \le x < r_n + 1/(10)^n$.

(a) Show that $f : (0,1) \to \{0,1,...,9\}^{\mathbb{N}}$ is a one-to-one map.

(b) Show that the set $S := \{f(x) : x \in (0,1)\}$ is uncountable.

(c) Conclude that $(0,1)$ is uncountable, and thus so is $\mathbb{R}$.

4. Find a $1-1$, onto map between $[0,1]$ and $(0,1)$.

5-8. Rudin p. 43 (Chapter 2.) Exercises 2, 3, 6, 7.