Math 131AH Winter 2018: Homework 1, Due 1/17

1. Show that the addition for natural numbers, as we defined in class, is commutative, i.e. prove that

$$(P_{n,m}) \quad m + n = n + m \text{ for all } m, n \in \mathbb{N}.$$

Hint: First prove $(P_{1,1})$, and then show $(P_{n,1})$, and then proceed to $(P_{n,m})$. You are allowed to use associative law on addition.

2. Let A and B be nonempty subsets of $\mathbb{R}^+ := \{x \in \mathbb{R} : x > 0\}$ which are bounded above. Let us define

$$C = \{xy : x \in A \text{ and } y \in B\}.$$

Show that $\sup C = (\sup A)(\sup B)$.

3. Let F be a field.

(a) Show that $(-x)(-y) = xy$ for $x, y \in F$. It is fine to follow the proof in the book but please clarify the properties that you use.

(b) Use this property to show that we always have $0 < 1$ in an ordered field F.

4-6. Rudin p22 (chapter 1) Exercise 3,5,9. Note that Exercise 3 yields another proof of Theorem 1.11 for ordered fields.