Math 131AH Winter 2018: Homework 8, Due 3/16

1. A subset A of a metric space X is *pathwise connected* if for every $x, y \in A$ there is a continuous function $f : [0,1] \to A$ such that f(0) = x and f(1) = y. Prove that every pathwise connected set is connected.

2. Let us consider

$$S := \{(x, \sin \frac{1}{x}) : 0 < x < 1\}$$
 and $E := \overline{S}$ in \mathbb{R}^2 .

- (a) Show that E is connected.
- (b) Show that E is not pathwise connected.
 - 3. Let (X, d) be a metric space.
- (a) Show that X is connected if and only if every continuous function $f: X \to \mathbb{N}$ is a constant.
- (b) Consider a continuous function $g: X \to Y$, where Y is another metric space. Using (a), show that if X is connected then g(X) is a connected subset of Y. In particular, if X is an interval in \mathbb{R} , then so is g(X).
 - 4-8. page 99, problem 6, 8, 18, 20, 24.