**Math 131AH Winter 2018: Homework 6, Due 2/28**

1. Show that the Cantor set is *totally disconnected* in $\mathbb{R}$, i.e. it contains no subset that is connected.

2. Show that a metric space $X$ is connected if and only if the following holds: the only subsets of $S$ which are both open and closed in $S$ are the empty set and $S$ itself.

3. Rudin Chapter 2 p44 problem 20.

4. Let $(s_n)_{n=1}^{\infty}$ be a bounded sequence in $\mathbb{R}$ (i.e. the set $\{s_n : n \in \mathbb{N}\}$ is bounded). Show that the set $S = \{x : x \text{ is a subsequential limit of } (s_n)\}$ is closed.

5. Let us define the metric space

$$X = \{\vec{a} = (a_1, a_2, a_3, ...) \in (\mathbb{R})^{\mathbb{N}} \text{ such that } \Sigma_{k=1}^{\infty} |a_k|^2 \leq 1\}.$$

with the metric $d(\vec{a}, \vec{b}) := [\Sigma_{k=1}^{\infty} |a_k - b_k|^2]^{1/2}$.

(a) Show that $(X, d)$ is a metric space.

(b) Show that $X$ is complete, bounded, but not compact.

6-8. Rudin p78, Problems 4,8,20.