Math 131A Winter 2018: Homework 7, Due 3/9

1-4. 19.1, 19.7, 19.9, 20.1. For 19.9 we accept the fact that $f(x) = \sin x$ is continuous in $\mathbb{R}$.

5. Let $f : [0, 1] \to \mathbb{R}$ be a continuous function with $f(0) = f(1)$. Show that there is some $x \in [0, 1/2]$ such that $f(x) = f(x + 1/2)$.

6. Show that $f(x) = \sqrt{x}$ is uniformly continuous in $[0, 1]$, using only the definition of the uniform continuity.

For 7.-8. S denotes a subset of $\mathbb{R}$.

7. (a) Show that if $f : S \to \mathbb{R}$ is uniformly continuous and if S is bounded, then $f(S)$ is bounded.
(b) Show also by a counterexample that the statement is false if f is only continuous.

8. Let $f : S \to \mathbb{R}$ be uniformly continuous and bounded. The function $\omega : (0, \infty) \to \mathbb{R}$ given by

$$\omega(\delta) := \sup\{|f(x) - f(y)| : x, y \in S, |x - y| < \delta\}$$

is called the *modulus of continuity* of f. Show that w is increasing and $\lim_{\delta \to 0^+} \omega(\delta) = 0$.