Math 131A Winter 2018: Homework 4, Due 2/9

1-3. 10.4, 10.7, 10.8

4. Let (s_n) be a sequence in $\mathbb{R}$, and suppose $\sup\{s_n : n \geq 1\} = \infty$. Show that $\limsup s_n = \infty$.

5. Let us consider the sequence $x_1 = 1$ and $x_{n+1} = 1 + \frac{1}{x_n}$.

(a) Show that $x_n \in [\frac{3}{2}, 2]$ for $n \geq 2$.

(b) Using (a), show that $|x_{n+1} - x_n| \leq \frac{4}{9}|x_n - x_{n-1}|$ for $n \geq 3$.

(c) Deduce that $\{x_n\}$ is Cauchy and thus it converges.

6. Suppose (s_n) does not have any subsequence that is monotone non-increasing. What can you say about s_n?

7-8. 11.5, 11.8.