Math 131A Winter 2018: Homework 5 Due 2/16

1. Let (a_n) and (b_n) be bounded sequences of real numbers. Show that

$$\limsup a_n + \liminf b_n \leq \limsup(a_n + b_n) \leq \limsup a_n + \limsup b_n.$$

Give an example of a single pair of sequences $(a_n), (b_n)$ for which both inequalities are strict.

2. Let (a_n) be a bounded sequence. Prove that there is exactly one real number L with the following two properties:

(i) For every $\varepsilon > 0$ there are only finitely many n for which $a_n > L + \varepsilon$;

(ii) For every $\varepsilon > 0$ there are infinitely many n for which $a_n > L - \varepsilon$.

Can you characterize this number L in terms of (a_n)?

3 .- 5. Ross 12.9, 12.10, 12.13.