Math 131A-Practice Midterm 1.

Please write clearly, and show your reasoning with mathematical rigor. You may use any correct rule about the algebra or order structure of $\mathbb{R}$ from Section 3 without proving it.

1.

(a) State the Principle of Mathematical Induction.

(b) Prove that for all positive integers n,

$$1 + 3 + ... + (2n - 1) = n^2.$$

2.

(a) State the Least Upper Bound Axiom (also called the Completeness Axiom).

(b) Let S be a non-empty subset of the real numbers such that S is bounded above. Let $M = \{y : y \geq x \text{ for all } x \in S.\}$. Prove that M is non-empty, that M is bounded below, and that

$$\sup S = \inf M.$$

3. Show that natural numbers do not have an upper bound (Do not use Archimedean Property).

4.

(a) Give the $\epsilon - N$ definition of $\lim_{n\to\infty} s_n = s$.

(b) Use the definition to prove that $\lim_{n\to\infty} \frac{10n+(-1)^n}{n} = 10$.

5. Carefully prove that if $s_n \to s$ and $t_n \to t$, where s and t are real numbers, then

$$s_n t_n \to st \text{ as } n \to \infty.$$

6.

(a) State the definition of $\limsup_{n\to\infty} s_n$ and $\liminf_{n\to\infty} s_n$ for a given sequence s_n.

(b) Show that $\liminf_{n\to\infty} s_n \leq \limsup_{n\to\infty} s_n$.