MATH 131A section 1: Midterm 2.

Please write clearly, and show your reasoning with mathematical rigor. You may use any correct rule about the algebra or order structure of $\mathbb{R}$ from Section 3 without proving it.

Name: Student ID:

Problem	Score
1	
2	
3	
4	
Total	

1. a) Suppose (s_n) is a sequence of real numbers satisfying $|s_n - s_{n+1}| \to 0$. Does this mean (s_n) converges? Either prove it or give a counter-example.

b) Now suppose that $|s_n - s_{n+1}| \leq \frac{1}{2^n}$. Show that (s_n) converges.

2. Let (a_n) be a sequence. Suppose that there is a subsequence $(a_{n_k})_{k\in I\!N}$ such that $\Sigma_{k=1}^{\infty} a_{n_k}$ converges. Show that then $\liminf |a_n| = 0$.

3. a) State the definition of continuity for a function f at $x_0 \in dom(f)$ (you can choose one of the two definitions given in the class).

b) Show that if $f : [0, 1] \to I\!R$ is a continuous function, then f is bounded in $[0, 1]$.

4. a) State the definition of uniform continuity for a function $f : S \to \mathbb{R}$ in S.

b) Show that if $f(x) = \sqrt{x}$ is uniformly continuous in $(0, 1]$. You need to prove that $\sqrt{x}$ is continuous at $x = 0$, if you wish to use the fact.