Math 131A Lecture 4: Homework 4, Due 5/6 in TA session

1. Let (s_n) be a sequence in $\mathbb{R}$.

(a) Show that if $\limsup s_n \leq 1$ if and only if the following holds:

$$\text{For every } \varepsilon > 0, \text{ there are only finitely many } n \text{ for which } s_n > 1 + \varepsilon.$$

(b) Suppose $\sup\{s_n : n \geq 1\} = \infty$. Show that $\limsup s_n = \infty$.

2. Let us consider the sequence $x_1 = 1$ and $x_{n+1} = 1 + \frac{1}{x_n}$.

(a) Show that $x_n \in [\frac{3}{2}, 2]$ for $n \geq 2$.

(b) Using (a), show that $|x_{n+1} - x_n| \leq \frac{4}{9}|x_n - x_{n-1}|$ for $n \geq 3$.

(c) Deduce that $\{x_n\}$ is Cauchy, and thus it converges.

3. 10.7

4. Suppose (s_n) does not have any subsequence that is monotone non-increasing. What can you say about (s_n)?

5-6. 11.8, 11.10.

7-11. 12.2, 12.4, 12.8, 12.10, 12.12