Math 131A Lecture 6: Homework 1, Due 4/8 in TA session

1. Section 1: 1.8.

2. Show that $n^4/4 < 1^3 + 2^3 + ... + n^3$ for $n \geq 1$ by induction.

3. Using induction, show that every integer $n > 1$ is either a prime or a product of primes.

4. Section 2:

(a) Show that $\sqrt{p}$, where p is a prime number, is not a rational number.

(b) Show that $p - q$, where p is a rational number and q an irrational number, is an irrational number.

5. Section 3: Ex. 3.4, 3.6

6. Let $(F, +, \cdot, \leq)$ be an ordered field with at least two elements. Let 0 and 1 denote the identities for addition and multiplication, respectively. Show that for two nonzero elements x, y of F such that $0 \leq x, y$, we have $x \leq y$ if and only if $x^2 \leq y^2$. Specify what axioms or theorems in the book you are using.