Q #:	Score:
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

Name:________________________ **PID:**________________________

TA:________________________ **Sec. No:**______ **Sec. Time:**____________

Math 20E Midterm 2

Summer Session II, 2015

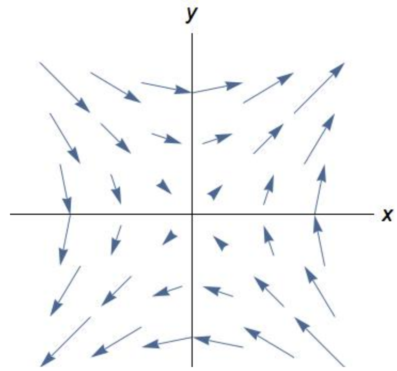
1. [10 points total]: Circle your answer to each of the following true/false or multiple-choice questions.

(a) [3 points]: Which of the following formulas defines the vector field $\vec{\mathbf{F}} : \mathbb{R}^2 \to \mathbb{R}^2$ represented in the figure?

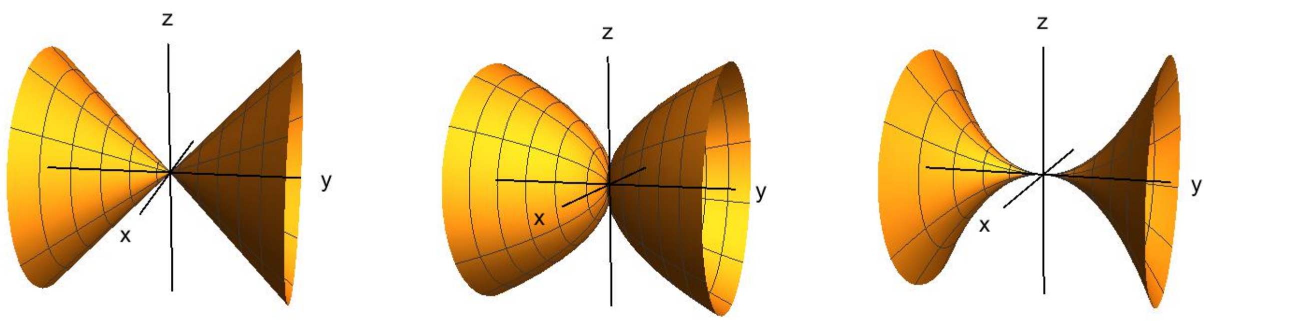
A. $\vec{\mathbf{F}}(x, y) = (x, -y)$

B. $\vec{\mathbf{F}}(x, y) = (y, x)$

C. $\vec{\mathbf{F}}(x, y) = (y, -x)$



(b) [3 points]: Which of the following shows the surface $x^2 - y^2 + z^2 = 0$?



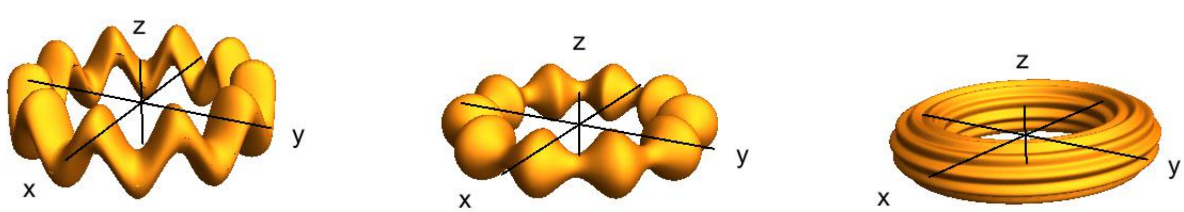
A **B** **C**

(c) [4 points]: Recall that a torus can be parametrized by

$$\Phi(\theta, \phi) = ((R + r\cos\theta)\cos\phi, (R + r\cos\theta)\sin\phi, r\sin\theta)$$

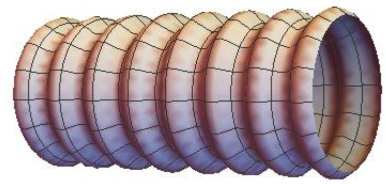
for (θ, ϕ) in $D = [0, 2\pi] \times [0, 2\pi]$ where r is the minor radius and R is the major radius. Which of the following pictures shows $S = \Phi(D)$ for

$$\Phi(\theta, \phi) = (\ (10 + (2 + \sin 10\phi)\cos\theta)\cos\phi, \quad (10 + (2 + \sin 10\phi)\cos\theta)\sin\phi, \quad (2 + \sin 10\phi)\sin\theta\ \ )?$$



A **B** **C**

2. [10 points total]: Consider the 15π-meter long storm drain pipe pictured and parametrized by



$$\Phi(y, \theta) = (\,(10 + \sin y)\cos\theta, \;\; y, \;\; (10 + \sin y)\sin\theta\,)$$

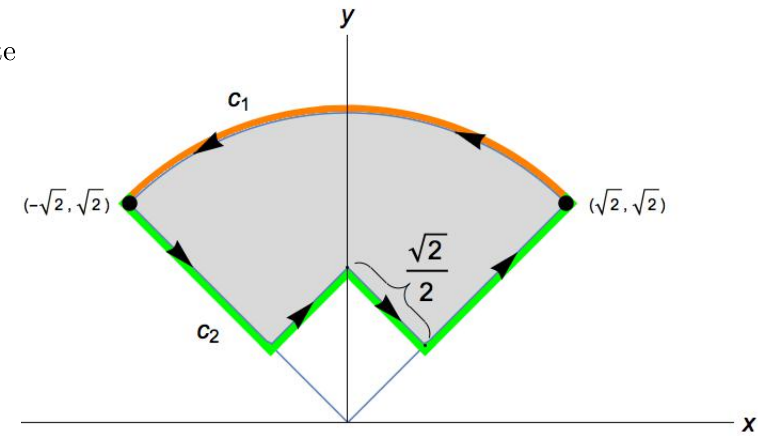
for (y, θ) in the domain $D = [0, 15\pi] \times [0, 2\pi]$.

(a) [4 points]: Calculate $\|\vec{\mathbf{T}}_y \times \vec{\mathbf{T}}_\theta\|$.

(b) [6 points]: Over the years, dirt accumulates on the surface S of the pipe. If the density of dirt at a point $(x, y, z) \in S$ is given by $f(x, y, z) = \sqrt{1 + \cos^2 y}$ kg/m^2, what is the total mass of dirt on the surface of the pipe in kilograms (kg)?

3. [10 points total]: Let D be the shaded portion of the baseball field pictured, which consists of square of side length $\frac{\sqrt{2}}{2}$ inside one quarter of the disc $x^2 + y^2 \leq 4$. Let $P(x, y) = x^2 - y$ and $Q(x, y) = x$.

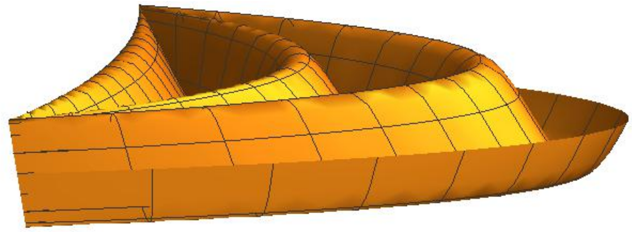
(a) [1 point]: Use geometric formulas to calculate the area of D.



(b) [4 points]: The outer arc $\mathbf{c_1}$ of the field is parametrized by $\mathbf{c_1}(t) = (2\cos t, 2\sin t)$ for $\frac{\pi}{4} \leq t \leq \frac{3\pi}{4}$. Evaluate $\int_{\mathbf{c_1}} P dx + Q dy$.

(c) [5 points]: Use Green's Theorem and your answers for parts **(a)** and **(b)** to evaluate $\int_{\mathbf{c_2}} P dx + Q dy$ where $\mathbf{c_2}$ is the part of ∂D not covered by $\mathbf{c_1}$, oriented as shown.
[If you did not solve **(a)** and/or **(b)**, you may use constants a and b in place of those answers]

4. [10 points total]: Let S be the surface parametrized by



$$\Phi(x, \theta) = (x, \theta(1 - x^2), \sin\theta)$$

for (x, θ) in the domain $D = [-1, 1] \times [0, 6\pi]$.

(a) [3 points]: Find $\vec{\mathbf{T}}_x \times \vec{\mathbf{T}}_\theta$.

(b) [7 points]: Let $\vec{\mathbf{F}}(x, y, z) = (x,\ x^2,\ z)$. Evaluate $\iint\limits_{\Phi} \vec{\mathbf{F}} \cdot d\vec{\mathbf{S}}$.

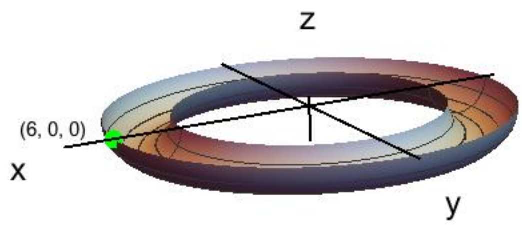
5. [10 points total]:

(a) [3 points]: Recall that a torus can be parametrized by

$$\Phi(\theta, \phi) = ((R + r\cos\theta)\cos\phi, (R + r\cos\theta)\sin\phi, r\sin\theta)$$

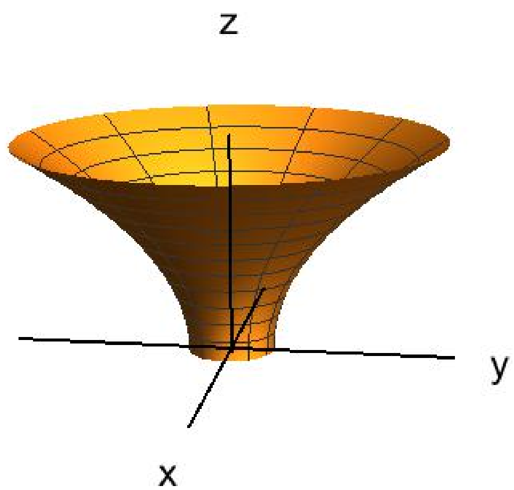
for (θ, ϕ) in $D = [0, 2\pi] \times [0, 2\pi]$ where r is the minor radius and R is the major radius.

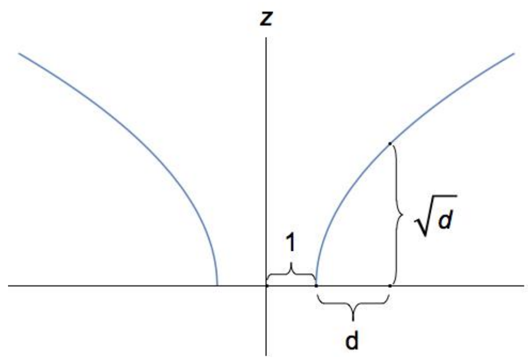
The pictured tray S consists of the half of a torus below the x-y plane for $r = 1$ and $R = 5$. Find a domain $D \subset \mathbb{R}^2$ and a parametrization $\Phi : D \to \mathbb{R}^3$ such that $S = \Phi(D)$.



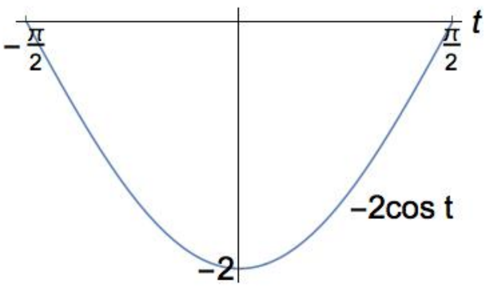
(b) [7 points]: Let S be the surface of the pictured birdbath for $0 \leq z \leq 4$. Horizontal cross-sections of the S are circles, the base of the birdbath being a circle of radius 1 in the x-y plane. At a distance d from the base of the birdbath, the birdbath has height $\sqrt{d}$ as shown below.

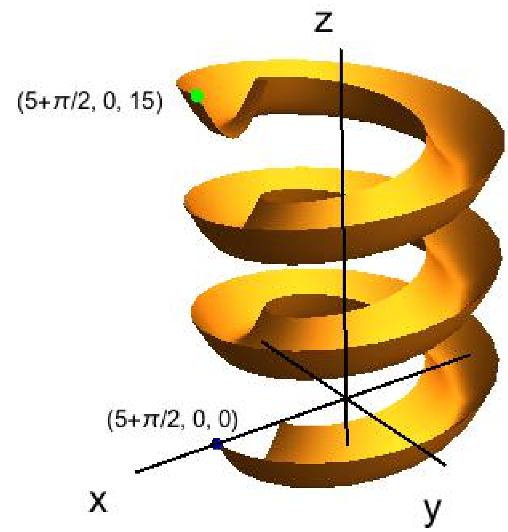
Find a domain $D \subset \mathbb{R}^2$ and a parametrization $\Phi : D \to \mathbb{R}^3$ such that $S = \Phi(D)$.





(c) [Extra Credit, 5 points]: The pictured slide S consists of a chute whose vertical cross-sections have the shape of the function $-2\cos t$ for $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.





The bottom of the chute is always a distant $R = 5$ from the z-axis, and the slide spirals around the z-axis at a constant incline, making 3 full revolutions as shown. Find a domain $D \subset \mathbb{R}^2$ and a parametrization $\Phi : D \to \mathbb{R}^3$ such that $S = \Phi(D)$.

[Scratch Paper]