Q #:	Score:
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

Name:_______________________ **PID:**_______________________

TA:_______________________ **Sec. No:**_____ **Sec. Time:**__________

Math 20E Midterm 1

Summer Session II, 2015

1. [10 points total]: Circle your answer to each of the following true/false or multiple-choice questions. [2 points each]:

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ be a function and $\vec{x_0} \in \mathbb{R}^n$.

(a) **True or False:** If f is differentiable at $\vec{x_0}$, then f must be continuous at $\vec{x_0}$.

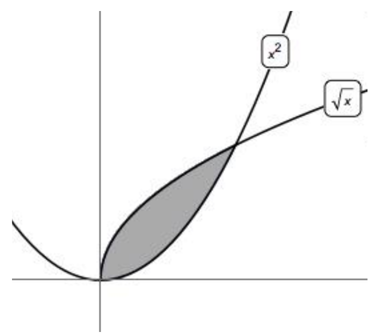
(b) **True or False:** If f is differentiable at $\vec{x_0}$, then all of the partial derivatives of f must exist at $\vec{x_0}$.

(c) **True or False:** If all of the partial derivatives of f exist at $\vec{x_0}$, then f must be differentiable at $\vec{x_0}$.

(d) Which of the following double integrals are evaluated over this shaded domain?

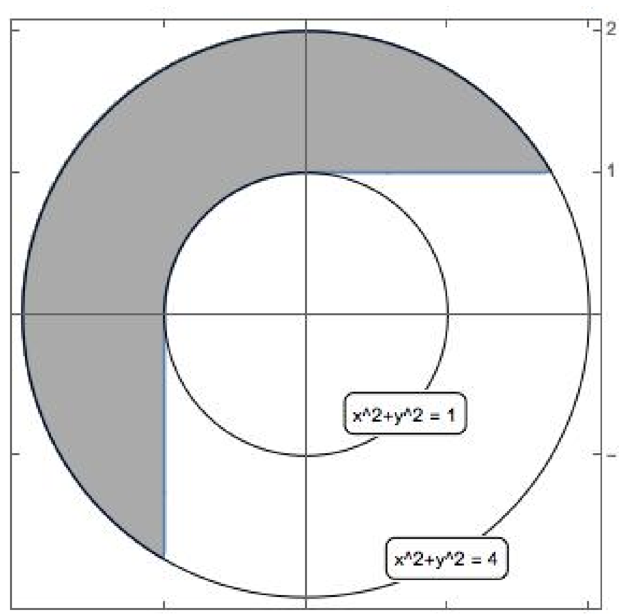
(i) $\int_0^1 \int_{x^2}^{\sqrt{x}} dydx$ (ii) $\int_0^1 \int_{\sqrt{y}}^{y^2} dxdy$ (iii) $\int_0^1 \int_{y^2}^{\sqrt{y}} dxdy$

A. (i) only
B. (ii) only
C. (iii) only
D. (i) and (ii)
E. (i) and (iii)



(e) The shaded region below is

A. x-simple
B. y-simple
C. both x-simple and y-simple
D. neither x-simple nor y-simple



2. [10 points total]:

(a) [2 points]: Let $g : \mathbb{R}^3 \to \mathbb{R}$ be defined by $g(x, y, z) = zx^2 + \cos y$. Find the matrix of partial derivatives $\mathrm{D}g(x, y, z)$.

(b) [3 points]: Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by $f(u, v) = (u \sin v, e^{u-v}, v)$. Find the matrix of partial derivatives $\mathrm{D}f(u, v)$.

(c) [2 points]: Find $f(1, 0)$ and $\mathrm{D}f(1, 0)$.

(d) [3 points]: Let $h : \mathbb{R}^3 \to \mathbb{R}$ be defined by $h = g \circ f$. Use the chain rule to find $\mathrm{D}h(1, 0)$.

3. [10 points]: Evaluate

$$\int_{1}^{e}\int_{\ln x}^{1} \frac{\sin(y^2)}{x}\, dy dx$$

4. [10 points]: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(u, v) = (u - e^v, u + e^v)$. Let $D^* = [0, 1] \times [0, 1]$ in the u-v plane. Calculate

$$\iint\limits_{T(D^*)} [x + y] \, dA$$

5. [10 points]: One day, Bill the baker spills an entire bag of sugar on his (2 meter)-by-(2 meter) baking table, represented by $D = [-1, 1] \times [-1, 1]$. Assume the *planar* density of sugar over a point (x, y) is given by

$$f(x, y) = x^2 + y^2 \quad \text{mg/m}^2$$

Luke the very lucky ant then eats his way across the table in path given by

$$c(t) = (e^{-t} \cos t, e^{-t} \sin t) \quad \text{from} \quad t = 0 \quad \text{to} \quad t = 10.$$

If the path Luke eats is 1 millimeter ($= \frac{1}{1000}$ meters) wide, then

$$[f(c(t)) \text{ mg/m}^2][\frac{1}{1000} \text{ m}] = \frac{f(c(t))}{1000} \text{ mg/m}$$

approximates the *linear* density of sugar along Luke's path. In mg, about how much sugar did Luke eat?