The Cave

In the year 2020, my mother discovers beautiful cave in northern Minnesota, which consists in part of a passage given by the surface S determined by

$$x = \frac{1}{\sqrt{y^2 + z^2}} \quad \text{for } 1 \leq x < \infty \text{ kilometers (km)}.$$

Later that year, the evil company Pollutacorp spills toxic waste in the area, and the walls of the cave passage are covered with waste. Assume the density of waste at a point $(x, y, z) \in S$ is

$$f(x, y, z) = \frac{y^2 + z^2}{x^2} \quad \text{tonnes/km}^2.$$

a) Sketch the cave. What is the total mass of toxic waste on the cave walls?

b) If my mother can remove the toxins at a rate of 10 tonnes/day, how long will it take her to restore the cave?

c) Show that the cave walls have infinite area. Ponder this paradox...

d) If my mother can restore cave walls at a rate of 1 km^2/day, how long will it take her to restore the cave?