Teaching with Technology: Increasing Proficiency in Computation and Enhancing Comprehension

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What matters most to you in your teaching?

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For many years now it has been clear that research, be it pure or applied, needs the computer as an essential tool. The solution of some of the "purest" problems -- the four-color problem, the classification of simple groups to mention only two – could not have been carried out without essential use of the computer. I can see the truth of this statement in my own research; the writing of many of my recent research papers began with computer experiments, although my research is not particularly in the field of applied mathematics.

A student who exits our undergraduate program without being conversant in computing will not be adequately prepared to do research or to work in industry.

Until the 1980s computing had to be done using the classical languages (Basic, Pascal, Fortran...). The simplest operations (finding roots of polynomials or matrix eigenvalues) required considerable programming, which sometimes got in the way of the mathematics. After 1980 this situation changed radically*; computer algebra systems* such as *Maple*, *Matlab* and *Mathematica* came into existence. These high level programming languages require only one-line commands to find polynomial roots and matrix eigenvalues, to do numerical integration or to produce book-quality graphs and plots. Programming is eliminated or kept to a minimum. Just as importantly, computer algebra systems do symbolic (not only numeric) computation, which extends their usefulness in an essential way. I teach my students how to use computer algebra systems to perform drudgery calculations, thereby allowing them to use their time to think about the essentials: theoretical concepts and problem solving.

How are you using technology to achieve your teaching goals?

All my undergraduate courses use *Mathematica*, a computer algebra system, available in various labs on campus.

Homework contains the traditional pencil-and-paper problems plus computer problems (about 50% each). The boundary between the two categories is not sharply defined. Students can use the computer to check the answers of their pencil and paper problems. They no longer have to depend on "answers-to-selected-problems" given in the last pages of a textbook. On the other hand, success in computing requires thinking, overall planning and strategy.

However, my use of the computer is not limited to furnishing the students with a useful tool. It is also used to enhance comprehension. To *give the definition of, and prove theorems* on a given mathematical concept (for example, matrix eigenvalues) is only the middle stage in a triad in a student's learning process. The first stage is *motivation* and the last stage is *applications.* After having learned the concept of eigenvalue in the comprehension stage, the students see its applications in the analysis of large oscillating structures. This analysis (which is impossible without the computer) makes eigenvalues "come to life" as descriptors of oscillating characteristics. Students can analyze changes in these characteristics as a consequence of changes in the design of the system. At this point eigenvalues become an indelible reality in the student's mind, rather than a mere dogmatic abstraction. The same paradigm applies to other mathematical ideas such as derivatives and integrals.

As to the actual use of technology in my courses, I communicate with students using a *class e-mail list.* This is a fast way of class contact by means of which I pass to the class handouts and class notes, homework solutions, last minute corrections, software tutorials, etc. (most of this material is posted also on the Web). I answer e-mail from students at various set times during the day, which, on broadband Internet, achieves "real-time" contact between the students and myself and saves students' time. Questions on the software are the easiest to answer, since commands are text-alone, but mathematical questions can be answered in the same way.

My e-mail contact with 31A students begins two-three weeks before the beginning of the course. Students are given the syllabus, the first class handout (containing the first two homeworks) and a software tutorial. The first couple of weeks of the quarter involve stress and uncertainty for entering freshmen, and some of this stress is relieved by advance knowledge of the material and of the course requirements.

Are existing textbooks adequate for your kind of teaching?

Hardly. In the area of undergraduate mathematics textbook publishers (and authors) have resorted to questionable practices for a long time. With a few honorable exceptions textbooks (especially lower division) are written to maximize sales, not quality. They are overpriced (sometimes by a factor of 2). New editions are put out with disconcerting frequency; the difference between an edition and the next is often negligible, but enough to force students to buy the new edition. The tables of contents don't seem to have changed for half a century, applications are often inane, and the problems unnatural and/or unchallenging. The computer revolution has been largely unnoticed; most textbooks incorporate a few "computer algebra problems" but these are merely an "add-on" with no organic relation to the text.

For the last fifteen years I have made all my courses textbook-independent. I use my own notes for all essential subjects, computing or theory. For the rest of the topics, calculus students are referred to an excellent textbook freely available on the Web. In some courses (e. g. Math 164, *Optimization*) my notes cover the entire course.

Are there negative reactions to teaching with technology?

Fear of (and resistance to) technology is as old as technology itself. In my high school days, we were punished if "caught" with a slide rule; in the words of one of my math professors, "the slide rule will destroy your sense of numbers". A distinguished historian of physics declared in the fifties that "computers mean the end of learning," and twenty years later, one of the best teachers in our Department boasted that "computers will be used in my course over my dead body". There was some basis in these pronouncements – in the fifties, one interfaced with computers using stacks of laboriously punched cards, and even in the seventies the simplest computations required considerable programming in computers that were not user-oriented. Nowadays, the interface human/computer has become so friendly and transparent (thanks in great part to the influence of the Apple Macintosh) that there are hardly any reasons to reject technology. Yet, arguments against technology still exist, not based on experience but on *a priori* unproven assumptions. One of them is, students will "unthinkingly accept computer results". The exact opposite is true; any computer result can (and should) be checked and double-checked in a variety of ways.

How have your students responded to your use of technology?

Favorably. Most of our undergraduates are very computer aware and have a clear idea of the importance of computing in their future careers. However, I don't assume that students have had previous computer experience; those that did not have it are given all necessary help to come up to speed. "Live" tutorials are given, where the students can execute given commands or modify these commands at will.

The most important effect of my use of computing on students has been stimulating their critical thinking and creativity. Frequently, after doing a computer problem students see the possibility of proving a theorem. This kind of "computer evidence" sometimes leads students to an actual result but sometimes it can be misleading. In either case, this process leads students to develop a strategy to plan computer experiments, to interpret their results using critical thinking and to ``look for patterns", that is, results that can be proved independently of the computer.

What new goals do you have for using technology in teaching?

Mathematics is a deductive science, and every theorem must be proved from first principles or from existing results. However, the mental processes leading to the formulation of new theorems depend on intuition and experimentation. Intuition may be innate (up to a point) but it can also be developed *from* experience and experimentation, much of it numerical, and must be combined with critical thinking. My main goal is to help students to develop the necessary combination of intuition and critical thinking.

This screed is a modified and updated version of an E-mail interview of May 2005 on the occasion of my being nominated for the first time to the Copenhaver Award for Innovation in Teaching with Technology.