Speaker: Johannes Sjöstrand, IMB Dijon.

Title: Tunneling for the $\overline{\partial}$-operator.

Abstract: This talk is based on a joint work with Martin Vogel. Let

$$P(x, hD_x; h) = p(x, hD_x) + hp_1(x, hD_x) + h^2 p_2(x, hD_x) + ...$$

be a differential operator in the semi-classical limit $0 < h \ll 1$. If the Poisson bracket

$$i^{-1}\{p, \overline{p}\}(x, \xi) = i^{-1}\left(p'_\xi \cdot \overline{p}'_x - p'_x \cdot \overline{p}'_\xi\right)(x, \xi)$$

does not vanish identically on the zero set of p, then P is non-self-adjoint and we know (cf. Hörmander 1960, ...) that P has singular values that are $\mathcal{O}(h^N)$, $\forall N \geq 0$, and even $\mathcal{O}(\exp -1/(Ch))$ for some $C > 0$ in the analytic case. Also, P has approximate null solutions concentrated to points in phase space where $p = 0$, $\{p, \overline{p}\}/i > 0$ and similarly P^* has approximate null solutions concentrated to points in phase space where $p = 0$, $\{p, \overline{p}\}/i < 0$.

In the analytic case, one may ask for the precise exponential decay rate of the small singular values. We obtain such results for the $\overline{\partial}$ operator on certain exponentially weighted L^2 spaces on a torus or on a cylinder, that can be interpretated in terms of quantum tunneling between the regions in the energy surface $p = 0$ where $\{p, \overline{p}\}/i > 0$ and $\{p, \overline{p}\}/i < 0$ respectively.