UNITARY SHIMURA VAREITY

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We give an example S of smooth Shimura variety for which irreducibility of the full Igusa tower is false but one can study the irreducible components explicitly. In other words, we construct a partial tower I°/S for which the axioms (A1–2) can be proved. As before, we prove that $S_{/\mathcal{W}}$ for sufficiently large discrete valuation ring $\mathcal{W}$ over $\mathbb{Z}_{(p)}$ is irreducible and smooth and that the Igusa tower $I_{/\mathbb{F}}$ is étale over $S_{/\mathbb{F}}$. Then for each point $x \in I(\mathcal{W})$, we take a coordinate $X_1, \dots, X_d$ of I and define a valuation v_x of the function field of I by $v_x(\sum_\alpha c(\alpha)X^\alpha) = \mathrm{Inf}_\alpha \mathrm{ord}_p(c(\alpha))$ ($X^\alpha = X_1^{\alpha_1} X_2^{\alpha_2} \cdots X_d^{\alpha_d}$). For any automorphism σ of $I_{/\mathcal{W}}$ fixing x, plainly $v_x \circ \sigma = v_x$. Then we conclude the irreducibility by showing that the stabilizers $\{D_x\}_{x \in I(\mathcal{W})}$ inside $Aut(I_{/\mathcal{W}})$ of $x \in I(\mathcal{W})$ covers sufficiently many conjugacy classes of tori enough to prove (A1–2).

1. Shimura varieties of unitary groups

We recall briefly the construction of the Shimura variety for unitary groups. The main source of the information is [PAF] Chapter 7. We fix an imaginary quadratic field field F. Suppose for simplicity that the fixed prime p is *split* in F. Write O for the integer ring of F.

1.1. **Unitary groups.**

Write c for the generator of $\mathrm{Gal}(F/\mathbb{Q})$ (the complex conjugation on F). We fix a vector space V over F with c-Hermitian alternating form $\langle\ ,\ \rangle : V \times V \to \mathbb{Q}$. We assume having an O-submodule $L \subset V$ of finite type such that

(L1) $L \otimes_{\mathbb{Z}} \mathbb{Q} = V$;
(L2) $\langle\ ,\ \rangle$ induces $\mathrm{Hom}_{\mathbb{Z}_p}(L_p, O_p) \cong L_p$, where $O_p = O \otimes_{\mathbb{Z}} \mathbb{Z}_p$ and $L_p = L \otimes_{\mathbb{Z}} \mathbb{Z}_p$.

We fix an O-lattice L of V as above.

We identify V with the column vector space F^r by fixing a base of V over F. Let $C = \mathrm{End}_F(V) = M_r(F)$. There exists an invertible matrix $s \in M_r(F)$ with ${}^t s^c = -s$ such that $\langle v, w\rangle = \mathrm{Tr}_{F/\mathbb{Q}}({}^t v s \cdot w^c)$, where $\mathrm{Tr}_{F/\mathbb{Q}}$ is the trace map $F \to \mathbb{Q}$. On C, we have the involution ι given by $x^\iota = s^{-1} {}^t x^c s$. Define algebraic groups defined over $\mathbb{Q}$ as functors from $\mathbb{Q}$-algebras R to groups:

$$\begin{aligned} GU(R) &= \left\{x \in C \otimes_{\mathbb{Q}} R \middle| xx^\iota \in R^\times\right\} \\ &= \left\{x \in C \otimes_{\mathbb{Q}} R \middle| {}^t x^c s \cdot x = \nu(x) s \ \text{ for }\ \nu(x) = xx^\iota \in R^\times\right\}, \\ U(R) &= \left\{x \in GU(R) \middle| xx^\iota = 1\right\},\ SU(R) = \left\{x \in U(R) \middle| \det(x) = 1\right\}, \end{aligned} \tag{1.1}$$

where $\det(x)$ is the determinant of x as an F-linear automorphism of V. Then SU is the derived group of GU and U. Let $Z \subset GU$ be the center. Then $Z(R) = (R \otimes_{\mathbb{Q}} F)^\times$.

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Since $F_{\mathbb{R}} = F \otimes_{\mathbb{Q}} \mathbb{R} = \mathbb{C}$ with $b^c = \overline{b}$ for complex conjugation $b \mapsto \overline{b}$, $S = \sqrt{-1}s \in M_r(F_{\mathbb{R}}) = M_r(\mathbb{C})$ is a Hermitian matrix. Thus $U(\mathbb{R})$ is the unitary group of S. We have $\mathrm{Hom}_{\mathrm{field}}(F, \mathbb{C}) = \{1, c\}$ for the identity inclusion 1. Writing the signature of S at $\sigma \in \Sigma$ as (m_1, m_c), we find $U(\mathbb{R}) \cong U_{m_1,m_c}(\mathbb{R}) = \left\{x \in GL_r(\mathbb{C}) \middle| {}^t\overline{x} I_{m_1,m_c} x = I_{m_1,m_c}\right\}$ for $I_{m_1,m_c} = \begin{pmatrix} 1_{m_1} & 0 \\ 0 & -1_{m_c} \end{pmatrix}$.

Exercise 1.1. Prove the following facts for any $\mathbb{Q}$-algebra R:

(1) If $s = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, then $\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)^\iota = \left(\begin{smallmatrix} \overline{d} & -\overline{b} \\ -\overline{c} & \overline{a} \end{smallmatrix}\right)$ and $SU(R) = SL_2(R)$,
$$GL_2(R) = \left\{x \in GU(R) \middle| \det(x) = \nu(x)\right\};$$

(2) $GU(\mathbb{Q}) = GL_2(\mathbb{Q})Z(\mathbb{Q})^\times$ and $GU(R) = GL_2(R)Z(R)$. (Hint: first show that $\zeta = \det(x)\nu(x)^{-1} \in (F \otimes_{\mathbb{Q}} R)^\times$ satisfies $\zeta\zeta^c = 1$; so, by Hilbert's theorem 90, $\zeta = \alpha\alpha^{-c}$ for $\alpha \in (F \otimes_{\mathbb{Q}} R)^\times$.)

1.2. **Abelian schemes of hermitian type.** To equip a complex structure with the real vector space $V_\infty = V \otimes_{\mathbb{Q}} \mathbb{R}$, we use an $\mathbb{R}$-algebra homomorphism $h : \mathbb{C} \hookrightarrow C_\infty = C \otimes_{\mathbb{Q}} \mathbb{R}$ with $h(\overline{z}) = h(z)^\iota$. We call such an algebra homomorphism *ι-homomorphism*. Then $h(i)^\iota = -h(i)$ for $i = \sqrt{-1}$ and hence $x^\rho = h(i)^{-1}x^\iota h(i)$ is an involution of C_∞.

Example 1.1. If $s = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, $h(a+bi) = \left(\begin{smallmatrix} a & -b \\ b & a \end{smallmatrix}\right) \in M_2(\mathbb{R}) \subset C_\infty$ is an ι-homomorphism.

We suppose

(pos) The symmetric real bilinear form $(v, w) \mapsto \langle v, h(i)w\rangle$ on V_∞ is positive-definite.

Exercise 1.2. Check h in Example 1.1 satisfies (pos).

By (pos), we have $0 < (xv, xv) = (v, (xx^\rho)v)$ for all $0 \neq v \in V_\infty$ and $x \in C_\infty$, and hence xx^ρ only has positive eigenvalues; so, ρ is a positive involution of C (i.e., $\mathrm{Tr}_{C/\mathbb{Q}}(xx^\rho) > 0$ unless $x = 0$).

Fix one such $h := h_{\mathbf{0}} : \mathbb{C} \to C_\infty$, and define $\mathfrak{X}$ (resp. $\mathfrak{X}^+$) by the collection of all conjugates of $h_{\mathbf{0}}$ under $GU(\mathbb{R})$ (resp. under $SU(\mathbb{R})$). Any two homomorphisms satisfying (pos) are conjugates under $SU(\mathbb{R})$ (see [PAF] Lemma 7.3). Thus $\mathfrak{X}^+ = SU(\mathbb{R})/C_{\mathbf{0}}$ for the stabilizer $C_{\mathbf{0}}$ of $h_{\mathbf{0}}$ in $SU(\mathbb{R})$ is connected and is a connected component of $\mathfrak{X}$. On $\mathfrak{X}$, $GU(\mathbb{R})$ acts by conjugation (from the left), and the stabilizer $C_{\mathbf{0}} \subset GU(\mathbb{R})$ of $h_{\mathbf{0}}$ is a maximal compact subgroup of $GU(\mathbb{R})$ modulo center by (pos).

Example 1.2. Assume that $s = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ and take $h_{\mathbf{0}}(a+bi) = \left(\begin{smallmatrix} a & -b \\ b & a \end{smallmatrix}\right)$. Since $h_{\mathbf{0}}(\mathbb{C}^\times)$ gives the stabilizer of $i \in \mathfrak{H} = \{z \in \mathbb{C} | \mathrm{Im}(z) > 0\}$, we have $\mathfrak{X}^+ \cong \mathfrak{H}$ by sending $gh_{\mathbf{0}}g^{-1}$ to $g(i)$.

Since $h : \mathbb{C} \to C_\infty$ is an $\mathbb{R}$-algebra homomorphism, we can split $V_{\mathbb{C}} = V \otimes_{\mathbb{Q}} \mathbb{C}$ into the direct sum of eigenspaces $V_{\mathbb{C}} = V_1 \oplus V_2$ so that $h(z)$ acts on V_1 (resp. V_2) through multiplication by z (resp. $\overline{z}$); thereby, we get a complex vector space structure on V_∞ by the projection $V_\infty \cong V_1$. Since $h(\mathbb{C}) \subset C_\infty$, $h(z)$ commutes with the action of F; so, V_j is stable under the action of $F_{\mathbb{C}} = F \otimes_{\mathbb{Q}} \mathbb{C}$. We get the representation $\rho_1 : F \hookrightarrow \mathrm{End}_{\mathbb{C}}(V_1)$. We define E for the subfield of $\mathbb{C}$ fixed by $\left\{\sigma \in \mathrm{Aut}(\mathbb{C}) \middle| \rho_1^\sigma \cong \rho_1\right\}$. If $h'(z) = g \cdot h(z)g^{-1}$ for $g \in GU(\mathbb{R})$, h' induces a similar decomposition $V_{\mathbb{C}} = V_1' \oplus V_2'$, and g induces a F-linear isomorphism between V_1 and V_1'; so, E is independent of the choice of h' in the

$GU(\mathbb{R})$-conjugacy class of h. This field E is called the *reflex field* of $(GU, \mathfrak{X})$ (and is a canonical field of definition of our canonical models of the Shimura variety).

Exercise 1.3. Prove $E = F$ or $\mathbb{Q}$ and that $E = \mathbb{Q}$ if and only if $m_1 = m_c$.

By the positivity (pos), the quotient complex torus $V_\infty/L = V_1/L$ has a Riemann form induced by $\langle\cdot,\cdot\rangle$. The theta functions with respect to the Hermitian form $\langle\cdot,\cdot\rangle$ gives global sections of an ample line bundle (e.g., [ABV] Chapter I) on V_1/L and hence embed V_1/L into a projective space over $\mathbb{C}$. The embedded image is the analytic space associated with an abelian variety $A_{h/\mathbb{C}}$ by Chow's theorem (see [ABV] page 33). Multiplication by $b \in O$ on V_1/L induces an embedding $i : O \hookrightarrow \mathrm{End}(A_{h/\mathbb{C}})$ and $i : F \hookrightarrow \mathrm{End}^{\mathbb{Q}}(A_{h/\mathbb{C}}) = \mathrm{End}(A_{h/\mathbb{C}}) \otimes_{\mathbb{Z}} \mathbb{Q}$.

The representation ρ_1 is given by the action of F on the tangent space $Lie(A_h) = V_1$ at the origin of A_h. Since A_h is projective, the field of definition of the abelian variety A_h is a field of finite type over $\mathbb{Q}$.

The reflex field E is the field of rationality of the representation of F on $Lie(A_h)$; so, the field of definition of (A_h, ι) always contains this field E. It would then be natural to expect that the moduli variety of triples (A, λ, ι) for an abelian variety A with F-linear isomorphism $Lie(A) \cong V_1$ is defined over E.

Since the isomorphism class of ρ_1 is determined by $\mathrm{Tr}(\rho_1)$ (see [MFG] Proposition 2.9), E is generated over $\mathbb{Q}$ by $\mathrm{Tr}(\rho_1(b))$ for all $b \in F$. We write O_E for the integer ring of E. Let $\mathbb{Z}_{(p)} = \mathbb{Z}_p \cap \mathbb{Q}$ and put $O_{(p)} = O \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)}$, $O_{E,(p)} = O_E \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)}$, and fix a valuation ring $\mathcal{V} \supset O_{E,(p)}$. The ring $\mathcal{V}$ has residue field $\mathbb{F}_p$ since p is split in E because $E \subset F$.

1.3. **Shimura variety for** GU**.** We study the classification problem of the following quadruples $(A, \lambda, i, \overline{\eta}^{(p)})_{/R}$: A is a (projective) abelian scheme over a base R, ${}^tA = \mathrm{Pic}^0_{A/R}(A)$ is the dual abelian scheme of A, $\lambda : A \to {}^tA$ is an isogeny with degree prime to p (prime-to-p isogeny) fiber-by-fiber geometrically induced from an ample divisor (polarization), $i : O_{(p)} \hookrightarrow \mathrm{End}_R^{\mathbb{Z}_{(p)}}(A) = \mathrm{End}_R(A) \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)}$ is a $\mathbb{Z}_{(p)}$-algebra embedding (taking 1 to the identity of A) with $\lambda \circ i(\alpha^c) = {}^t i(\alpha) \circ \lambda$ for all $\alpha \in O$, and $\eta^{(p)}$ is a level structure. Regarding ${}^tA$ as a left O-module by $O \ni b \mapsto {}^t i(b^c) \in \mathrm{End}({}^tA)$, λ is F-linear. Hereafter we call λ *F-linear* in this sense. The base scheme R is assumed to be a scheme over $Spec(\mathcal{V})$.

We clarify the meaning of the level structure $\eta^{(p)}$. Fix a base (geometric) point $s \in R$ and write A_s for the fiber of A at s. We consider the Tate module $\mathcal{T}(A_s) = \varprojlim_N A[N](k(s))$ and $V^{(p)}(A_s) = \mathcal{T}(A_s) \otimes_{\mathbb{Z}} \mathbb{A}^{(p\infty)}$, where N runs over all positive integers ordered by divisibility. The prime-to-p level structure $\eta^{(p)} : V(\mathbb{A}^{(p\infty)}) = V \otimes_{\mathbb{Q}} \mathbb{A}^{(p\infty)} \cong V^{(p)}(A_s)$ is an O-linear isomorphism. The duality pairing $e_N : A[N] \times {}^tA[N] \to \mu_N$ composed with λ gives, after taking the limit with respect to N, an alternating form $(\cdot,\cdot)_\lambda : V^{(p)}(A_s) \times V^{(p)}(A_s) \to \mathbb{A}^{(p\infty)}(1) := \lim_{p \nmid N} \mu_N$ satisfying the following conditions:

(P1) $(\alpha(x), y)_\lambda = (x, \alpha^c(y))_\lambda$ for $\alpha \in \mathrm{End}(A_{/B})$;

(P2) The pairing induces the self-duality: $A[p^n] \cong \mathrm{Hom}(A[p^n], \mu_{p^n})$ if $N = p^n$.

We require that $\eta^{(p)}$ send the alternating form $\langle\cdot,\cdot\rangle$ to $(\cdot,\cdot)_\lambda$ up to the scalar multiple by $(\mathbb{A}^{(p\infty)})^\times$. This is possible, because $\mathbb{A}^{(p\infty)}(1) \cong \mathbb{A}^{(p\infty)}$ up to scalar in $(\mathbb{A}^{(p\infty)})^\times$. Then $\eta^{(p)}$ is required to be an isomorphism of skew Hermitian F-modules with respect to the pairing $\langle\cdot,\cdot\rangle_\lambda$ on $V^{(p)}(A_s)$.

The algebraic fundamental group $\pi_1(R,s)$ acts on $V^{(p)}(A_s)$ leaving the skew Hermitian form $\langle\cdot,\cdot\rangle_\lambda$ stable up to the scalar (because it keeps the Weil e_N-pairing; see [ABV] Section 20). Take a closed subgroup $K^{(p)} \subset GU(\mathbb{A}^{(p\infty)})$. We write $\overline{\eta}^{(p)}$ for the orbit $\eta^{(p)} \circ K^{(p)}$. If $\sigma \circ \overline{\eta}^{(p)} = \overline{\eta}^{(p)}$ for all $\sigma \in \pi_1(R,s)$, we say the level structure $\overline{\eta}^{(p)}$ is defined over R. Even if we change the point $s \in R$, everything will be conjugated by an isomorphism; so, the definition does not depend on the choice of s as long as R is connected. For nonconnected R, we choose one geometric point at each connected component.

We call a quadruple $\underline{A}_{/R} = (A,\lambda,i,\overline{\eta}^{(p)})_{/R}$ isomorphic to $\underline{A}'_{/R} = (A',\lambda',i',\overline{\eta}'^{(p)})_{/R}$ if we have an O-linear isogeny $\phi : A \to A'$ defined over R such that $p \nmid \deg(\phi)$, $\phi^*\lambda' = {}^t\phi \circ \lambda' \circ \phi = \nu\lambda$ with $\nu \in \mathbb{Z}^\times_{(p)+}$, $\phi \circ i \circ \phi^{-1} = i'$, and $\overline{\eta}'^{(p)} = \phi \circ \overline{\eta}^{(p)}$. Here $\mathbb{Z}^\times_{(p)+}$ is the collection of all positive elements in $\mathbb{Z}^\times_{(p)}$. Thus ϕ brings the prime-to-p polarization class $\overline{\lambda}' = \{\nu\lambda' | \nu \in \mathbb{Z}^\times_{(p)+}\}$ of λ' to the class $\overline{\lambda}$ of λ: $\phi^*\overline{\lambda}' = \overline{\lambda}$. In this case, we write $A \approx A'$. We write $A \cong A'$ if the isogeny is an isomorphism of abelian schemes; that is, $\deg(\phi) = 1$.

We take the fibered category $\mathcal{C} = \mathcal{C}_{F,V}$ of the quadruples $(A,\lambda,i,\eta^{(p)})_{/R}$ over the category $\mathcal{V}$-SCH of $\mathcal{V}$-schemes and define

$$\text{(1.2)}\quad \mathrm{Hom}_{\mathcal{C}_{/R}}((A,\lambda,i,\eta^{(p)})_{/R},(A',\lambda',i',\eta'^{(p)})_{/R}) = \left\{\phi \in \mathrm{Hom}_R(A,A') \otimes_{\mathbb{Z}} \mathbb{Z}_{(p)} \middle| \begin{matrix} {}^t\phi \circ \lambda' \circ \phi = \nu\lambda \ \text{ with } \ 0 < \nu \in \mathbb{Z}^\times_{(p)+}, \\ \phi \circ i = i' \circ \phi \ \text{ and } \ \eta'^{(p)} = \phi \circ \eta^{(p)} \end{matrix}\right\}.$$

The representation ρ_1 is well defined over $\mathcal{V}$, since p splits in F; so, it is well defined over $\mathcal{O}_R$ for any $\mathcal{V}$-scheme R. We consider the functor $\mathcal{E}^{(p)} : \mathcal{V}\text{-}SCH \to SETS$ given by

$$\mathcal{E}^{(p)}(R) = \left\{\underline{A}_{/R} = (A,\overline{\lambda},i,\eta^{(p)})_{/R} \middle| Lie(A) \cong \rho_1 \text{ over } \mathcal{O}_R\right\}/\approx .$$

Since $A_{/R}$ is a group scheme, its tangent space at the zero section has a Lie algebra structure over $\mathcal{O}_R$. We write $Lie(A)$ for this Lie algebra. Since A is smooth over R, $Lie(A)$ is a locally free $\mathcal{O}_R$-module of rank $\dim_R A$. In our case, for a given quadruple $\underline{A} = (A,\lambda,i,\overline{\eta}^{(p)})_{/R}$, the Lie algebra $Lie(A)$ of A over $\mathcal{O}_R$ is an $O_{(p)}$-module via i. Since $Lie(A)$ is locally free of rank $\dim_R A$ over $\mathcal{O}_R$, we can think of an isomorphism $Lie(A) \cong \rho_1$ of $\mathcal{O}_R$-representations of $O_{(p)}$. One can find in [PAF] Chapter 7 a proof of the following theorem due to Shimura, Delighne and Kottwitz.

Theorem 1.1. *The functor $\mathcal{E}^{(p)}$ is representable by a quasi-projective smooth pro-scheme $Sh^{(p)}$ over $\mathcal{V}$. Letting $g \in GU(\mathbb{A}^{(p)})$ act on $Sh^{(p)}$ by $\eta^{(p)} \mapsto \eta^{(p)} \circ g$, for any open compact subgroup $K \subset G(\mathbb{A}^{(p\infty)})$, the quotient scheme $Sh^{(p)}_K = Sh^{(p)}/K$ exists as a quasi-projective scheme of finite type over $\mathcal{V}$, and $Sh^{(p)} = \varprojlim_K Sh^{(p)}_K$. The Shimura variety $Sh^{(p)}_K$ is projective over $\mathcal{V}$ if the Hermitian pairing $\langle\cdot,\cdot\rangle$ is anisotropic.*

For a finite set of primes Σ containing p and ∞, we can think of the Shimura variety away from Σ as follows. Write $\Sigma = \{p,\infty\} \sqcup \Sigma'$ and we assume that $\Sigma' \neq \emptyset$. Let $K_\Sigma = \{g \in G(\mathbb{Q}_{\Sigma'}) | gL_{\Sigma'} = L_{\Sigma'}\}$ and $Sh^{(\Sigma)} = Sh^{(p)}/K_\Sigma$. It is known that $Sh^{(\Sigma)}_{/\mathcal{V}}$ is a smooth (quasi-projective) pro-scheme.

Fix an embedding $i_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$ so that $i_p = 1$ on F. Let $\mathcal{W}$ be the pull back by i_p of the p-adic integer ring of the maximal unramified extension of $\mathbb{Q}_p$; so, $\mathcal{V} \subset \mathcal{W}$. Write $\mathcal{K}$ be the filed of fraction of $\mathcal{W}$. Let $Sh^{(\Sigma)}_{/\mathcal{W}} = Sh^{(\Sigma)} \times_{\mathrm{Spec}(\mathcal{V})} \mathrm{Spec}(\mathcal{W})$. By the reduction map, we have $\pi_0(Sh^{(\Sigma)}_{/\mathcal{K}}) \cong \pi_0(Sh^{(\Sigma)}_{/\mathbb{F}})$ for $Sh^{(\Sigma)}_{/\mathbb{F}} = Sh^{(\Sigma)}_{/\mathcal{W}} \times_{\mathcal{W}} \mathbb{F}$ by Zariski's connectedness theorem and the existence of smooth toroidal compactification of $Sh^{(p)}_{K/W}$ ($W = \varprojlim_n \mathcal{W}/p^n\mathcal{W}$), and $SU(\mathbb{A}^{(\Sigma)})$ fixes each element of $\pi_0(Sh^{(\Sigma)}_{/\mathcal{K}})$ because $\mathfrak{X}^+$ is a quotient of $SU(\mathbb{R})$. Thus

Proposition 1.2. *Geometrically irreducible components of* $Sh^{(\Sigma)}_{/\mathcal{W}}$ *are irreducible over* $\mathcal{W}$*, and the group* $SU(\mathbb{A}^{(\Sigma)})$ *leaves stable each irreducible component of* $Sh^{(\Sigma)}_{/\mathbb{F}} = Sh^{(\Sigma)}_{/\mathcal{W}} \times_{\mathcal{W}} \mathbb{F}$.

We can compute the stabilizer in $GU(\mathbb{A}^{(\Sigma)})$ of each point of $\pi_0(Sh^{(\Sigma)}_{/\mathbb{F}})$ explicitly ([H06] Lemma 1.1).

REFERENCES

[ABV] D. Mumford, *Abelian Varieties*, TIFR Studies in Mathematics, Oxford University Press, New York, 1994.

[C] C.-L. Chai, Methods for p-adic monodromy, preprint 2006, (available at `http://www.math.upenn.edu/~chai`)

[DAV] G. Faltings and C.-L. Chai, *Degeneration of Abelian Varieties*, Springer, New York, 1990

[GIT] D. Mumford, *Geometric Invariant Theory*, Ergebnisse **34**, Springer, New York, 1965.

[MFG] H. Hida, *Modular Forms and Galois Cohomology*, Cambridge Studies in Advanced Mathematics **69**, Cambridge University Press, Cambridge, England, 2000.

[PAF] H. Hida, *p-Adic Automorphic Forms on Shimura Varieties*, Springer Monographs in Mathematics, 2004, Springer

[H05] H. Hida, p-Adic automorphic forms on reductive groups, Astérisque **298** (2005), 147–254

[H06] H. Hida, Irreducibility of the Igusa tower, preprint, 2006

[IAT] G. Shimura, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton University Press, Princeton, NJ, and Iwanami Shoten, Tokyo, 1971.

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