Adjoint modular Galois representations and their Selmer groups

Haruzo Hida*, Jacques Tilouine and Eric Urban

The talk at the conference on Elliptic Curves and Modular Forms at the National Academy of Science was presented by the first named author. This note is a summary of the talk. The purpose of the talk was to describe several formulas giving the characteristic ideal of the Selmer group of the Galois representation as in the title in terms of their *L*-values. We fix a prime $p \geq 5$. Although we can treat the general case of modular Galois representations ramifying at finitely many primes and ∞, to keep the paper short within the limit, we assume that the ramification is concentrated in $\{p,\infty\}$.

Although each author had already worked out some of their share of the work presented here before they visited the Mehta Research Institute of Mathematics and Mathematical Physics (MRI, Allahabad, India) in January-February, 1996, much of the coordination in bringing all the efforts into a general frame work was done while they were visiting Allahabad. We are grateful to Professor Dipendra Prasad at MRI for giving us the opportunity of working together and to the audience at MRI for patiently listening to our lectures on the subject whose formulation was not yet definite.

1. Selmer groups. Let G be the Galois group of the maximal extension $\mathbf{Q}^{(p)}/\mathbf{Q}$ unramified outside $\{p,\infty\}$. Let O be a valuation ring finite flat over $\mathbf{Z}_p$ with residue field $\mathbf{F}$. We start with a 2-dimensional continuous representation $\varphi : G \to GL_2(A)$ for a complete (noetherian) local O-algebra A with residue field $\mathbf{F} = A/m_A$. The power series ring $O[[T_1,\dots,T_r]]$ is an example of such A. For a subfield F of $\mathbf{Q}^{(p)}$, consider

(AI_F) *The restriction of* $\varphi \bmod m_A$ *to* $H = \mathrm{Gal}(\mathbf{Q}^{(p)}/F)$ *is absolutely irreducible.*

We let G act on $V = A^2$ via φ and let G acts on $\mathrm{End}(V)$ by conjugation:

$$x \mapsto \varphi\otimes\varphi^{\vee}(\sigma)x = \varphi(\sigma)x\varphi(\sigma)^{-1}$$

and look at its 3 dimensional factor $Ad(\varphi) : G \to GL_3(A)$ acting on trace zero subspace $V(Ad(\varphi)) = \{x \in \mathrm{End}(V) \mid \mathrm{Tr}(x) = 0\}$. Thus $\varphi\otimes\varphi^{\vee} = Ad(\varphi)\oplus\mathbf{1}$. To guarantee the irreducibility of $Ad(\varphi)$ mod m_A, we assume (AI_F) for the unique quadratic extension $F = \mathbf{Q}(\sqrt{(-1)^{(p-1)/2}p})$. We further assume the following two conditions:

---

* Talk at NAS conference on March 16, 1996 at Washington D.C., and the first named author is partially supported by an NSF grant.

(Ord) *For each decomposition group* D *over* p, $\varphi|_D \cong \begin{pmatrix} \delta & * \\ 0 & \varepsilon \end{pmatrix}$ *with unramified* δ;

(Reg) $$\delta \bmod m_A \neq \varepsilon \bmod m_A.$$

We then write $V(\delta) \subset V$ for the δ-eigen-subspace, and for each A-submodule X of $V(Ad(\varphi))$, let $X^* = X\otimes_A A^*$ for the Pontryagin dual $A^* = Hom_O(A,\mathbf{Q}_p/\mathbf{Z}_p)$ of A. We put $V_+ = \{\phi \in V(Ad(\varphi)) \subset End(V) \mid \phi(V(\delta)) = 0\}$. Then we define the Selmer group for $Ad(\varphi)$, as a special case of Greenberg's definition [G] (see also [H96]):

$$Sel(Ad(\varphi)) = Ker(H^1(G,V(Ad(\varphi))^*) \to H^1(I,V(Ad(\varphi))^*/V_+^*))$$

for the inertia subgroup I of D. This is obviously a generalization of the class group; for example, taking a quadratic character χ of G,

$$Sel(\chi) = Ker(H^1(G,V(\chi)^*) \to H^1(I,V(\chi)^*))$$

is the p-class group of the quadratic extension F fixed by $Ker(\chi)$. Thus if A = O and if $L(1,Ad(\varphi)) \neq 0$, a naive guess is that $Sel(Ad(\varphi))$ is finite and that its order is the p-part of $L(1,Ad(\varphi))$ up to the transcendental factor. The finiteness is first shown by Flach [F] and then by Wiles [W]. We discuss later some good cases where this guess works well. We generalize above definition to a tensor product $Ad(\varphi)\otimes\varepsilon$ with a character $\varepsilon : G \to B^\times$, replacing A by $A\hat{\otimes}_O B$, V_+ by $V_+(Ad(\varphi)\otimes\varepsilon) = V_+\hat{\otimes}B$:

$$Sel(Ad(\varphi)\otimes\varepsilon) = Ker(H^1(G,V(Ad(\varphi)\otimes\varepsilon)^*) \to H^1(I,V(Ad(\varphi)\otimes\varepsilon)^*/V_+(Ad(\varphi)\otimes\varepsilon)^*)),$$

which is a discrete module over $A\hat{\otimes}_O B$.

2. Elliptic curve over Q. Suppose that φ_0 is the Galois representation on $H^1(E_{/\overline{\mathbf{Q}}},\mathbf{Z}_p)$ for a modular elliptic curve $E_{/\mathbf{Q}}$. In addition to our assumption, we assume that E has multiplicative reduction at p and has good reduction outside p. This is just to simplify the presentation. Thus we can embed E into the jacobian $J = J_0(p)$ of the modular curve $X_0(p)$. Taking the dual of the inclusion $E \subset J$, we have a quotient map $\pi : J \to E$. Then $J = E+A$ for $A = Ker(\pi)$ and $E \cap A$ is a finite group of square order. For a Néron differential ω on the Néron model $E_{/\mathbf{Z}}$, by a result of Mazur [M] Corollary 4.1, we may assume that $\pi^*\omega = 2^e(2\pi i f_0(z)dz)$ for a primitive form $f_0 \in S_2(\Gamma_0(p))$ and $e \in \mathbf{Z}$. Choosing a base $c_\pm$ of $\pm$-eigenspace of $H_1(E(\mathbf{C}),\mathbf{Z})$ under complex conjugation, we define $\Omega_\pm$ by $\int_{c_\pm}\omega$ after normalizing $c_\pm$ as described below. The following formula was proven 15 years ago in [H81]:

(IN1) $$\frac{L(1,\mathrm{Ad}(\varphi_0))}{C^{-1}(2\pi i)\Omega_+\Omega_-} = \sqrt{|E\cap A|} \in \mathbf{Z} \quad \text{(Intersection number formula)},$$

where $C = 2^{a+2e}p(p-1)$ for $2^a = [H_1(E(\mathbf{C}),\mathbf{Z}):\mathbf{Z}c_+\oplus\mathbf{Z}c_-]$. We define the canonical period $U(f_0)$ of f_0 by $C^{-1}(2\pi i)\Omega_+\Omega_-$. In [H81], to get (IN), we used the period determinant

$$u = \left|\det\begin{pmatrix}\int_{c_1}\omega & \int_{c_2}\omega \\ \int_{c_1}\overline{\omega} & \int_{c_2}\overline{\omega}\end{pmatrix}\right|$$

for a $\mathbf{Z}$-base $\{c_1,c_2\}$ of $H_1(E(\mathbf{C}),\mathbf{Z})$ in place of $\Omega_+\Omega_-$ (see [H81] (6.20b)). Writing $\omega_\pm = \frac{1}{2}(\omega\pm\overline{\omega})$, we see $\int_{c_\pm}\omega = \pm\int_{\gamma_\pm}\omega_\pm$, and thus $\Omega_+ \in \mathbf{R}$ and $\Omega_- \in \sqrt{-1}\mathbf{R}$. Replacing c_+ and c_- by their negative if necessary, we may assume that $\Omega_+ > 0$ and $\sqrt{-1}\Omega_- > 0$. Under this normalization, the formula (IN1) is correct. Then by definition, $2^a u = \sqrt{-1}\Omega_+\Omega_-$, and we can deduce the formula (IN1) from [H81] Theorem 6.1 by just remarking that $L^*_{f_0}/L_{f_0} \cong E\cap A$ under the notation of the theorem quoted.

In [H81], actually a formula similar to (IN1) is proven for the modular Galois representation attached to any holomorphic primitive form of weight ≥ 2. The formula is generalized later to cohomological cusp forms on $GL(2)$ over imaginary quadratic fields in [U95].

Let H be the subalgebra of $\mathrm{End}(J)$ generated by Hecke operators $T(n)$. Then π induces the projection $\lambda : H \to \mathbf{Z} \subset \mathrm{End}(E)$ and another projection $\lambda' : H \to \mathrm{End}(A)$. Then we define two finite modules:

$$C_0 = \mathrm{Im}(\lambda)\otimes_H \mathrm{Im}(\lambda') \quad\text{and}\quad C_1 = \Omega_{H/\mathbf{Z}}\otimes_{H,\lambda}\mathrm{Im}(\lambda) = \mathrm{Ker}(\lambda)/\mathrm{Ker}(\lambda)^2.$$

It is proven in [H88] (5.8b) that

$$(E\cap A)_p \cong (C_{0,p})^2 \quad \text{as } H\text{-modules}.$$

Note that $\mathrm{Spec}(C_0)$ is the scheme theoretic intersection of $\mathrm{Spec}(\mathrm{Im}(\lambda))$ and $\mathrm{Spec}(\mathrm{Im}(\lambda'))$ in $\mathrm{Spec}(H)$. Thus we get

(IN2) $$\text{p-part of } \frac{L(1,\mathrm{Ad}(\varphi_0))}{C^{-1}(2\pi i)\Omega_+\Omega_-} = |C_{0,p}| \quad \text{(Intersection number formula in Spec(H))}.$$

Recently, R. Taylor and A. Wiles ([W] and [TW]) have shown that $|C_{0,p}| = |C_{1,p}|$, and A. Wiles [W] has shown, using Mazur's deformation theory of Galois representations (cf. [MT]),

$$C_{1,p} \cong \mathrm{Sel}(\mathrm{Ad}(\varphi_0)),$$

which is a key to his proof of Fermat's last theorem. Thus under various assumptions on p we made, we finally get a formula for the order of $\mathrm{Sel}(\mathrm{Ad}(\varphi_0))$:

$$\text{(CN1)} \qquad \text{p-part of } \frac{L(1,\mathrm{Ad}(\varphi_0))}{C^{-1}(2\pi i)\Omega_+\Omega_-} = |\mathrm{Sel}(\mathrm{Ad}(\varphi_0))|.$$

3. One variable case. The cusp form $f_0 \in S_2(\Gamma_0(p))$ can be lifted to a p-adic family of p-ordinary common eigenforms $f_k = \sum_{n=1}^{\infty} a(n;f_k)q^n \in S_{k+2}(\Gamma_0(p),\omega^{-k})$ $(k \geq 0)$ for the Teichmüller character ω (cf. [H] Chapter 7, Theorem 7.3.4). For this, we actually need to fix an embedding $i_p : \overline{\mathbf{Q}} \subset \overline{\mathbf{Q}}_p$. Then "p-ordinarity" of f_k implies that the q-expansion coefficient of f_k in q^p satisfies $|a(p;f_k)|_p = 1$. Note that, by multiplicative reduction hypothesis, $a(p;f_0) = \pm 1$. This family yields a Galois representation $\varphi : G \to GL_2(\Lambda)$ for a finite flat $O[[T]]$-algebra Λ ([H] Section 7.5). For simplicity, we assume $\Lambda = O[[T]]$. Then writing the specialization of φ via $1+T$ a u^k for $u = 1+p$ as φ_k, φ_k is the Galois representation of the cusp form f_k. Then the Pontryagin dual $\mathrm{Sel}^*(\mathrm{Ad}(\varphi))$ of $\mathrm{Sel}(\mathrm{Ad}(\varphi))$ is shown by Wiles and Taylor to be a torsion $O[[T]]$-module of finite type, and its characteristic power series is given by the characteristic power series of the Λ-adic congruence module $C_{0,\Lambda}$. To define $C_{0,\Lambda}$, we need to introduce the space S_Λ of p-ordinary Λ-adic cusp forms. For that, we consider the subspace $S_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}})$ of $S_{k+2}(\Gamma_0(p),\omega^{-k})$ made of cusp forms f with $a(n;f) \in \overline{\mathbf{Q}}$ for all n. We consider the $\overline{\mathbf{Q}}_p$-span $S_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}}_p)$ of $S_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}})$ in $\overline{\mathbf{Q}}_p[[q]]$ via q-expansion. We write $S^{ord}_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}}_p)$ for the subspace of $S_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}}_p)$ spanned by all p-ordinary eigenforms. An element $F \in S_\Lambda$ is a formal q-expansion $\sum_{n>0} a_n(T)q^n \in \Lambda[[q]]$ such that the specialization F_k via $1+T$ a u^k is the q-expansion of an element in $S^{ord}_{k+2}(\Gamma_0(p),\omega^{-k};\overline{\mathbf{Q}}_p)$ for all $k \geq 0$. In particular, we have a unique F such that $F_k = f_k$ for all $k \geq 0$. Then S_Λ is free of finite rank over Λ on which Hecke operators $T(n)$ naturally act ([H] Section 7.3). Let H be the Λ-subalgebra of $\mathrm{End}_\Lambda(L)$ generated by $T(n)$ for all n. Then $F|h = \lambda(h)F$ defines a Λ-algebra homomorphism $\lambda : H \to \Lambda$. We also have $\lambda' : H \to \mathrm{End}_\Lambda(\mathrm{Ker}(\lambda))$ given by multiplication by $h \in H$ on $\mathrm{Ker}(\lambda)$. Then we define

$$C_{0,\Lambda} = \mathrm{Im}(\lambda)\otimes_H \mathrm{Im}(\lambda') \text{ and } C_{1,\Lambda} = \Omega_{H/\Lambda}\otimes_{H,\lambda}\mathrm{Im}(\lambda) = \mathrm{Ker}(\lambda)/\mathrm{Ker}(\lambda)^2.$$

Then it is easy to see that $C_{0,\Lambda} \cong \Lambda/(\eta(T))$ for an element $\eta(T) \in \Lambda$. We can deduce from the result of Wiles and Taylor in [W] Theorem 3.3 and [TW] that

$$(\eta(T)) = \mathrm{char}_\Lambda(C_{1,\Lambda}) \text{ and } C_{1,\Lambda} \cong \mathrm{Sel}^*(\mathrm{Ad}(\varphi)).$$

Here the characteristic ideal $\mathrm{char}_A(M)$ for a torsion A-module of finite type M over a normal noetherian ring A is given by the product of prime divisors P in A with exponent given by $\mathrm{length}_{A_P}M_P$ of the localization M_P at P. Note that, as shown in [H88] Theorem 0.1, for a canonical period $U(f_k)$ associated to f_k,

(CL2) $$\eta(u^k-1) = \frac{L(1,\mathrm{Ad}(\varphi_k))}{U(f_k)} \text{ up to p-adic units.}$$

This formula is not completely satisfactory, because the p-adic L-function $\eta(T)$ is determined only up to units in Λ. For Λ-adic forms of CM type, we can choose a suitable Katz p-adic L-function in place of η ([T88-89], [MT90], [HT94]). In general, we can only make a conjecture predicting the existence of a canonical p-adic L-function $L_p(\mathrm{Ad}(\varphi))$ with precise interpolation property [H96a], which generates $\mathrm{char}_\Lambda(\mathrm{Sel}^*(\mathrm{Ad}(\varphi))) = (\eta(T))$ after extending scalar to the p-adic integer ring O_Ω of the p-adic completion Ω of the algebraic closure $\overline{\mathbf{Q}}_p$ of $\mathbf{Q}_p$.

3. Two variable case. Now we look at the universal character $\nu : G \to O[[S]]^\times$ deforming the identity character of G. Writing $\mathbf{Q}_\infty$ for the cyclotomic $\mathbf{Z}_p$-extension of $\mathbf{Q}$ and $\Gamma = \mathrm{Gal}(\mathbf{Q}_\infty/\mathbf{Q})$, the tautological character: $\Gamma \subset O[[\Gamma]]$ induces the above ν for $S = \gamma-1$ for a generator γ of Γ. Then we consider $\mathrm{Sel}^*(\mathrm{Ad}(\varphi)\otimes\nu^{-1})$, which is a module over $O[[T,S]]$ of finite type. Recently we have proven a control theorem for $\mathrm{Sel}(\mathrm{Ad}(\varphi)\otimes\nu^{-1})$ giving

Theorem 1 (H. Hida [H96b]). *The module* $\mathrm{Sel}^*(\mathrm{Ad}(\varphi)\otimes\nu^{-1})$ *is a torsion* $O[[T,S]]$*-module of finite type. Moreover, the characteristic power series of* $\mathrm{Sel}^*(\mathrm{Ad}(\varphi)\otimes\nu^{-1})$ *is of the form* $S\Psi(T,S)$ *in* $O[[T,S]]$ *and* $\Psi(T,0) \mid \eta(T)\frac{da}{dT}(T)$ *in* $O[[T]]$, *where* $a(T)$ *is the eigenvalue of* $T(p)$ *for* F *lifting* f_0.

The details of the proof will be published in [H96b]. In early 1980's, we constructed in [H90] a two variable p-adic L-function $L(T,S)$ in $\eta(T)^{-1}O[[T,S]]$ such that for even m with $-k \le m \le 0$,

$$\eta(u^k-1)L(u^k-1,u^m-1) = *E(k,m)\frac{L(1-m,\mathrm{Ad}(\varphi_k))}{(2\pi i)^{-2m}U(f_k)}$$

for a factor E like an Euler p-factor and a simple constant $*$. This L-function ηL again has ambiguity by units in Λ, although $L(T,S)$ is uniquely determined. In [H96a], the existence of a canonical p-adic L-functions $L_p(\mathrm{Ad}(\varphi)\otimes\nu^{-1})$ in $O[[T,S]]$ (for $\mathrm{Ad}(\varphi)\otimes\nu^{-1}$) with precise interpolation property is conjectured. In particular, we should have an equality: $L(T,S) = \dfrac{L_p(\mathrm{Ad}(\varphi)\otimes\nu^{-1})}{L_p(\mathrm{Ad}(\varphi))}$. Anyway the denominator and the numerator are not yet known to exist in general in spite of the known existence of the ratio $L(T,S)$. Because of this, we need to use $\eta(T)$ as a replacement of $L_p(\mathrm{Ad}(\varphi))$.

The Euler factor E vanishes on the line $m = 0$. Thus we can write $\eta(T)L(T,S) = S\Phi(T,S)$. On the automorphic side of p-adic L-functions, very recently, Greenberg and Tilouine have proven the following limit formula:

Theorem 2 (R. Greenberg and J. Tilouine [GT96]). *We have* $\Phi(0,0) = \eta(0)\frac{da}{dT}(0)$ *up to units in* O.

By the theorem of St. Etienne [BDGP] (due to four people: Barré-Sirieix, Diaz, Gramain, and Philibert at Université de St. Etienne, France), $\frac{da}{dT}(0) \neq 0$. Thus if one can prove the divisibility: $\Phi \mid \Psi$ in $O[[T,S]]$, the following conjecture follows:

Main Conjecture. *We have* $\Phi = \Psi$ *up to a unit in* $O[[T,S]]$.

Actually this conjecture is close to be proven (but not yet completely) [U96]. Let us explain Urban's strategy. First of all, there is a theory of nearly p-ordinary $O[[T,S]]$-adic forms on $GSp(4)$, developed mainly by Tilouine and Urban [TU95-96], and gluing Weissauer's modular Galois representations, Urban associated to each $O[[T,S]]$-adic eigen cusp forms G a Galois representation $\rho_G : G \to GSp(4;I)$ for a finite extension I of $O[[T,S]]$ with explicitly given characteristic polynomials of Frobeniai outside p. This construction is a non-trivial endeavor, although gluing looks easy, because in $GSp(4)$-case, residual Galois representations are often reducible. We look at the Klingen-style $O[[T,S]]$-adic Eisenstein series E induced from the Λ-adic form F. The Galois representation ρ_E attached to E is of the following form:

$$\rho_E \cong \begin{pmatrix} \varphi & * \\ 0 & {}^t\varphi^{-1}\otimes\nu \end{pmatrix} \subset GSp(4;I).$$

Note here that φ is self dual up to $\det(\varphi)$, and hence ρ_E has values in $GSp(4)$. The constant term of E at the non-standard parabolic subgroup P is almost equal to F times $\eta(T)L(T,S)$. Here we mean by "non-standard" the parabolic subgroup given by:

$$P = \left\{ \begin{pmatrix} * & 0 & * & * \\ * & * & * & * \\ * & 0 & * & * \\ 0 & 0 & 0 & * \end{pmatrix} \in GSp(4) \right\}.$$

Thus the Eisenstein ideal giving congruence between E and another $O[[T,S]]$ cusp form G should be generated by $\eta(T)L(T,S)$. In particular, Urban has shown for such Eisenstein primes P dividing $\Phi(T,S)$, if $G \equiv E \bmod P$ for a cusp form G, ρ_G is irreducible, and hence, we conclude P divides Ψ by a $Gsp(4)$-version of an argument of Wiles in [W1] applied to $GL(2)$, assuming that ρ_G is nearly p-ordinary (that is, the image of D is in a Borel subgroup). The representation ρ_G has densely populated specializations on $\mathrm{Spec}(I)$ which are crystalline at p and belongs to a compatible system of Galois representations. Thus we have two characteristic polynomials at p of each crystalline specialization. One is that of the crystalline Frobenius $L_{cris}(X)$, and the other $L_{et}(X)$ is that of the Frobenius at p of non-p-adic member of the compatible system. This near p-ordinarity follows if one can prove $L_{cris}(X) = L_{et}(X)$ for densely populated specializations, which is a standard conjecture and is known to be true at least for constant sheaves (that is, so to speak, weight 0 specilization). In order to establish the congruence $G \equiv E \bmod P^e$ for $P|\Phi$ with maximal possible exponent e, we need to have precise information of E (not just its existence), like that of Fourier coefficients of non-constant terms and its Whittaker model.

References.

[BDGP] K. Barré-Sirieix, G. Diaz, F. Gramain, and G. Philibert, Une preuve de la conjecture de Mahler-Manin,
Inventiones Math. 124 (1996), 1--9

[F] M. Flach, A finiteness theorem for the symmetric square of an elliptic curve, Invent. Math. 109 (1992), 307-327

[G] R. Greenberg, Iwasawa theory and p-adic deformation of motives, Proc. Symp. Pure Math. 55 (1994) Part 2, 193-223

[GT96] R. Greenberg and J. Tilouine, in preparation

[H] H. Hida, *Elementary Theory of L-functions and Eisenstein series,* 1993, LMSST 26, Cambridge University Press

[H81] H.Hida, Congruences of cusp forms and special values of their zeta functions, Invent. Math. vol.63 (1981), 225-261

[H88] H. Hida, Modules of congruence of Hecke algebras and *L*-functions associated with cusp forms, Amer. J. Math. vol.110 (1988), 323-382

[H90] H. Hida, p-adic L-functions for base change lifts of GL_2 to GL_3, Proc. of Conference on "Automorphic forms, Shimura varieties, and *L*-functions, Michigan, Ann Arbor, July, 1988, Perspectives in Math. 11 (1990), 93-142

[H96a] H. Hida, On the search of genuine p-adic modular *L*-functions for GL(n), Mémoires de Societe Mathematique de France 67, 1996

[H96b] H. Hida, On Selmer groups of adjoint modular Galois representations, to appear in Sém. Théorie de Nombre, Paris, 1993-94, in the lecture notes series of LMS, Cambridge University Press, 1996

[HT94] H. Hida and J. Tilouine, On the anti-cyclotomic main conjecture for CM fields, Inventiones Math. 117 (1994), 89-147

[M] B. Mazur, Rational isogenies of prime degree, Invent. Math. 44 (1978), 129-162

[MT90] B. Mazur and J. Tilouine, Représentations galoisiennes, différentielles de Kähler et «conjectures principales». Publ. I.H.E.S. 71 (1990), 65-103

[TW] R. Taylor and A. Wiles, Ring theoretic properties of certain Hecke modules, Ann. of Math. 142 (1995), 553-572

[T88] J. Tilouine, Théorie d'Iwasawa classique et de l'algèbre de Hecke ordinaire, Compositio Math. 65 (1988), 265-320

[T89] J. Tilouine, Sur la conjecture principale anticyclotomique, Duke Math. J. 59 (1989), 629-673

[TU95] J. Tilouine and E. Urban, Familles p-adiques à trois varaiables de formes de Siegel et de représentations galoisiennes, C. R. Acad. Sci. Paris 321 series I (1995), 5-10

[TU96] J. Tiouine and E. Urban, Several variable *p*-Adic families of Siegel-Hilbert cucp eigensystems and their Galois representations, preprint

[U95] E. Urban, Formes automorphes cuspidales pour GL_2 sur un corps quadratiques imaginaire. Valeurs spéciales de fonctions L et congruences, Compositio Math. 99 (1995), 283-324

[U96] E. Urban, Lectures at the Mehta Research Institute for Mathematics and Mathematical Physics, Allahabad, India, 1996

[W] A. Wiles, Modular elliptic curves and Fermat's last theorem, Ann. of Math. 142 (1995), 443-551

[W1] A. Wiles, The Iwasawa conjecture for totally real fields, Ann. of Math. 131 (1990), 493-540