Correction to:
[H96]: On Selmer Groups of Adjoint Modular Galois Representations
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The proof given in [H96] contains a gap stemming from a mis-statement of the assertion of Proposition 1.1 in [H96]. Here we would like to give a description of valid assertions of [H96] and would like to correct false statements there. Page and line numbers quoted here are the ones in the file attached (those numbers in parentheses are page and line numbers in the paper published)

We correct the statements of Proposition 1.1, Theorems 3.2 and 3.3 of [H96] and give a corrected proof of them along the line employed in [H96]. We use the notation introduced in [H96]. Here is the corrected statement of Proposition 1.1 in [H96]:

Proposition 0.1. *Suppose the surjectivity of θ and μ. Then we have the following canonical exact sequence of H–modules:*

$$Tor_1^H(B, \mathrm{Ker}(\mu)) \longrightarrow C_1(\theta; T) \otimes_T B \longrightarrow C_1(\lambda; B) \longrightarrow C_1(\mu; B) \longrightarrow 0.$$

In [H96], the first term of the above exact sequence is written as $Tor_1^T(B, \mathrm{Ker}(\mu))$. The proof given in [H96] gives the correct result without any change. The mis-statement of this result affects the assertions made at several other places of [H96]. Here is the corrected statement of Theorem 3.2 of [H96]:

Theorem 0.2. *Suppose* $(\mathrm{AI}_{\mathbb{Q}})$*, the conditions of $\mathcal{D}$ for $\overline{\rho}$ and that $\mathbb{I}$ is a torsion-free Λ_0–module of finite type giving the normalization of an irreducible component of $Spec(R_{\mathbb{Q}})$. Let $\mathrm{Sel}^*_{\mathcal{L}}(\mathrm{Ad}(\varphi) \otimes \nu^{-1})_{/\mathbb{Q}}$ be the Pontryagin dual module of the Selmer group $\mathrm{Sel}_{\mathcal{L}}(\mathrm{Ad}(\varphi) \otimes \nu^{-1})_{/\mathbb{Q}}$. We have the following two exact sequences of $\mathbb{I}$–modules:*

$$\mathbb{I} \otimes_{\mathbb{Z}_p} \Gamma_j \xrightarrow{\varepsilon_j} \frac{\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi) \otimes \nu^{-1})_{/\mathbb{Q}}}{(\gamma^{p^j} - 1)\,\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi) \otimes \nu^{-1})_{/\mathbb{Q}}} \longrightarrow C_1(\lambda_j; \mathbb{I}) \longrightarrow 0$$

$$C_1(\pi_\infty; \mathbb{I}_0) \otimes_{\mathbb{I}_0} \mathbb{I} \xrightarrow{\iota_\infty} \mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi) \otimes \nu^{-1})_{/\mathbb{Q}} \longrightarrow \mathbb{I} \longrightarrow 0.$$

Moreover suppose that $R_{\mathbb{Q}}$ is reduced and either that $R_{\mathbb{Q}}$ is a Λ–module of finite type or that $Spec(\mathbb{I}_0)$ is an irreducible component of $Spec(R_{\mathbb{Q}})$. Then ε_j is injective.

In the original version in [H96], it is claimed that $\mathrm{Ker}(\iota_\infty)$ is a pseudo-null $\mathbb{I}[[\Gamma]]$–module, which does not immediately follow from the method employed in [H96]. Thus the analysis of $\mathrm{Ker}(\iota_j)$ given from the line 10 from the bottom of page 17 (page 105) of [H96] to the line 16 from the bottom of page 18 (page 106) does not stand as it is. Removing this part from the proof, we get the corrected assertion.

We also need to correct the assertion of Theorem 3.3 in [H96]. Here is the corrected one:

Theorem 0.3. *Suppose* $(\mathrm{AI}_{\mathbb{Q}})$, (Ind)*, that $\mathbb{I}$ is a torsion-free Λ–module of finite type giving the normalization of an irreducible component of $Spec(R_{\mathbb{Q}})$ and that $\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi))_{/\mathbb{Q}}$ is a torsion $\mathbb{I}$–module. Then we have*

(i) $\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi)) \otimes \nu^{-1})_{/\mathbb{Q}}$ *is a torsion $\mathbb{I}[[\Gamma]]$–module of finite type;*

(ii) *There is a pseudo-isomorphism of* $\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi)) \otimes \nu^{-1})_{/\mathbb{Q}}$ *into* $M \times \mathbb{I}$ *for a torsion* $\mathbb{I}[[\Gamma]]$*–module* $M = C_1(\lambda'_\infty; \mathbb{I})$ *such that* $M/(\gamma - 1)M$ *is a torsion* $\mathbb{I}$*–module;*

(iii) *If* $\mathrm{Sel}^*_{\mathcal{M}'}(\mathrm{Ad}(\varphi))_{/\mathbb{Q}}$ *is a pseudo-null* $\mathbb{I}$*–module and* $\Lambda' = \mathbb{I}$*, then* $\mathrm{Sel}^*_{\mathcal{M}}(\mathrm{Ad}(\varphi)) \otimes \nu^{-1})_{/\mathbb{Q}}$ *is pseudo isomorphic to* $\mathbb{I}$*, on which* Γ *acts trivially;*

(iv) *If* $\mathbb{I}_0$ *is formally smooth over* $\mathfrak{O}$*, then we have the following exact sequence of* $\mathbb{I}[[\Gamma]]$*–modules :*

$$0 \longrightarrow C_1(\pi_\infty; \mathbb{I}) \longrightarrow C_1(\lambda'_\infty; \mathbb{I}) \longrightarrow \widehat{\Omega}_{\mathbb{I}/\Lambda'} \longrightarrow 0,$$

where $\widehat{\Omega}_{\mathbb{I}/\Lambda'}$ *is the module of continuous* 1*–differentials or equivalently is the* $m_{\mathbb{I}}$*–adic completion of* $\Omega_{\mathbb{I}/\Lambda'}$ *(which is a torsion* $\mathbb{I}$*–module of finite type by* (Ind)*).*

Originally M is claimed to be pseudo isomorphic to $C_1(\pi_\infty; \mathbb{I}_0) \otimes_{\mathbb{I}_0} \mathbb{I}$ in the assertion (ii). This is true if $\mathbb{I}_0$ is formally smooth over $\mathfrak{O}$, and in this case, M is isomorphic to $C_1(\pi_\infty; \mathbb{I}_0) \otimes_{\mathbb{I}_0} \mathbb{I}$; otherwise, the proof given there does not immediately show the pseudo-isomorphism. The two arguments given after Theorem 3.3 in [H96] proving the control of $C_1(\lambda'_j; \mathbb{I})$ and the $\mathbb{I}[[\Gamma]]$–torsion-ness of $C_1(\lambda'_\infty; \mathbb{I})$ are correct. However, the argument from the line 19 of page 21 (page 109) of [H96] to the line 3 from the bottom of the same page, relating $C_1(\lambda'_\infty; \mathbb{I})$ and $C_1(\pi_\infty; \mathbb{I}) \otimes_{\mathbb{I}_0} \mathbb{I}$ up to $\mathbb{I}[[\Gamma]]$–pseudo null modules, is incorrect. The result holds when $\mathbb{I}$ is formally smooth as later proved in [H96] pages 24-25 (pages 112-113). To recover the result (ii), we need to show that

$$0 \longrightarrow Y \longrightarrow C_1(\lambda_\infty; \mathbb{I}) \longrightarrow C_1(\lambda'_\infty; \mathbb{I}) \longrightarrow 0$$

is exact for an $\mathbb{I}[[\Gamma]]$–torsion module Y. This can be done as follows: Note that

$$C_1(\lambda'_j; \mathbb{I}) \cong \widehat{\Omega}_{R_j/\Lambda'_j} \otimes_{R_j} \mathbb{I} \quad \text{and} \quad C_1(\lambda_j; \mathbb{I}) \cong \widehat{\Omega}_{R_j/\Lambda_j} \otimes_{R_j} \mathbb{I}.$$

We have by definition $\Lambda_\infty \cong \mathfrak{O}$ and $\Lambda'_\infty \cong \mathfrak{O}[[X]]$ by (Ind) of [H96] page 19 (page 107). Then the exact sequence:

$$\mathbb{I} \cong \widehat{\Omega}_{\mathfrak{O}[[X]]/\mathfrak{O}} \otimes_{\mathfrak{O}[[X]]} \mathbb{I} \longrightarrow \widehat{\Omega}_{R_\infty/\Lambda_\infty} \otimes_{R_\infty} \mathbb{I} \longrightarrow \widehat{\Omega}_{R_\infty/\Lambda'_\infty} \otimes_{R_\infty} \mathbb{I} \longrightarrow 0$$

shows that Y is the image of $\mathbb{I}$, which is a torsion $\mathbb{I}[[\Gamma]]$–module. In this way, we can recover the assertion of Theorem 3.3 in [H96] as stated above.

Here we list minor mistakes and misprints in [H96]:

page and line	read	should read
page 4 (page 92) (Ext2):	$Tor_1^{B'}$	$Tor_1^{T'}$ for $T' = T \otimes_A B$
page 12 (page 100) line 5 (line 6):	$T = R_E \otimes_{\Lambda'_E} \mathbb{I}$	$T = R_F \otimes_{\Lambda'_F} \mathbb{I}$
page 17 (page 105) (Ext5-6):	$Tor_1^{T_j}$	$Tor_1^{T_k}$
page 28 (page 116) lines 7 from the bottom (lines 13):	$c(h\tau)c(\tau)$	$c(h\tau) = \pi(h)c(\tau)$

page 39 (page 127) Proposition A.2.3: Remove $R_G^{\chi, ord}$ from the statement.