Lemma 3.17 in the μ-invariant paper [M] :Ann. of Math. **172** (2010) (July 24, 2022):

Lemma 0.1. *Let $N_i = A$ for a commutative ring A $(i = 1, 2, \ldots, m)$. Let $N \subset N_1 \times N_2 \times \cdots \times N_m = A^m$ be an A-free submodule of A^m with $m \geq 2$. If A is a product of finitely many local rings and the projection of N to $N_i \times N_m$ is surjective for all $i = 1, 2, \ldots, m-1$ and the projection π' of N to $N' := N_1 \times N_2 \times \cdots \times N_{m-1}$ is surjective, we have $N = A^m$.*

has to be replaced by

Lemma 0.2. *Let $N_i = A$ for a commutative ring A $(i = 1, 2, \ldots, m)$. Let $N \subset N_1 \times N_2 \times \cdots \times N_m = A^m$ be an A-free submodule of A^m with $m \geq 2$. Suppose:*

(1) *A is a product of finitely many local rings;*

(2) *the projection of N to $N_i \times N_m$ is surjective for all $i = 1, 2, \ldots, m-1$;*

(3) *the projection π' of N to $N' := N_1 \times N_2 \times \cdots \times N_{m-1}$ is surjective.*

Identifying $N' \subset N$ by $N' \cong N' \times \{0\}$, either we have $N = A^m$ or $N' \cap N$ satisfies the three conditions (1)–(3) for $m-1$ in place of m.

Proof. We may assume that A is a local ring. For an A-module, we write $\overline{X} := X \otimes_A k$ for the residue field k of A. Since all projections of N to N_i is surjective and N_i is A-free, tensoring k over A preserves intersections; i.e., $\overline{X} \cap \overline{Y} = \overline{X \cap Y}$ for $X, Y = N_i, N, N'$ and so on. Tensor product also preserves surjections (i.e., left exact), we may assume that A is a field k. We have a short exact sequence:

$$0 \to N \cap N' \to N \to N_m \to 0.$$

If the intersection $N \cap (N' \times 0) \cong k^{m-1}$, we have $\dim_k N = m$ and $N = k^m$.

Assume that $N \cap (N' \times 0)$ has dimension $< m-1$. Since $N = N' \oplus N_m$, N' is embedded into $N_1 \times N_2 \times \cdots \times N_{m-1}$. Identifying N' with its image in $N_1 \times N_2 \times \cdots \times N_{m-1}$, $N' \cap N$ satisfies the three conditions (1)–(3) for $m-1$ in place of m. □

This lemma fits well with the induction in the first proof of [M,Corollary 3.19] without much modification as the case $m = 2$ is taken care of by Proposition 3.15 and Corollary 3.16 in [M] directly.