EXTRA HOMEWORK EXERCISES NO.2

1. Homework set No.3 (in addition to Exercises in the notes)

The due date of this third set is May 30, 2008.

(1) Consider the closed subscheme $X = \mathrm{Proj}\left(\frac{\mathbb{C}[X,Y,Z]}{(Y^2Z-Z^3+X^3)}\right)$ in $\mathbf{P}^2(\mathbb{C}) = \mathrm{Proj}(\mathbb{C}[X,Y,Z])$. How many points are there in $X(\mathbb{C}) - D_+(Z)$? Justify your answer.

(2) Let X be as above. Compute $\Gamma(X, \Omega_{X/\mathbb{C}})$.

(3) Compute $\Gamma(\mathbf{P}^2, \Omega_{\mathbf{P}^2/\mathbb{C}})$.

(4) Give an example of the "non-surjective" natural map $\widetilde{M} \otimes_{\widetilde{A}} \otimes \widetilde{N} \xrightarrow{\lambda} (\widetilde{M \otimes_A N})$. Justify your answer. Hint: Consider $A = \mathbb{C}[g_1, g_2]$ (a two variable polynomial ring) but $\deg(g_1) = 1$ and $\deg(g_2) = 2$. Take $M = N = A(1)$.

(5) Is $\mathrm{Proj}(\mathbb{C}[g_1, g_2])$ as above isomorphic to $\mathbf{P}^1_{/\mathbb{C}}$? Justify your answer.

(6) Let $A = \bigoplus_n A_n$ be a graded $\mathbb{Q}$-algebra. For a $\mathbb{Q}$-algebra R, consider $\varphi : A \to A \otimes_{\mathbb{Q}} R$ given by $\varphi(a) = a \otimes 1$. Prove that $D_+(\varphi(a)) \cong D_+(a) \times_{\mathrm{Spec}(\mathbb{Q})} \mathrm{Spec}(R)$.

(7) For A as above, take two graded A-modules M and N. Does $M_m \otimes_{A_0} N_n$ (the tensor product over "A_0") inject naturally into $M \otimes_A N$? Justify your answer.

(8) Let A be a B-algebra, and put $S = \mathrm{Spec}(A)$. Consider the B-algebra homomorphism

$$m : A \otimes_B A \to A$$

sending $a \otimes a'$ to aa'. Show that the morphism $m^* : S \to S^2 := \mathrm{Spec}(A) \times_{\mathrm{Spec}(B)} \mathrm{Spec}(A)$ corresponding to m sends $x \in S(R)$ to the diagonal $(x, x) \in S^2(R)$.

(9) Glue two copies of $\mathbb{G}_a = \mathrm{Spec}(\mathbb{C}[t])$ and $\mathrm{Spec}(\mathbb{C}[s])$ by identifying $t = s$ on $\mathrm{Spec}(\mathbb{C}[t, t^{-1}]) \cong \mathrm{Spec}(\mathbb{C}[s, s^{-1}])$ getting a scheme X. Is the diagonal image $X \cong \Delta_X \subset X \times X$ closed? Justify your answer.

(10) The same question as above for $X = \mathbf{P}^1_{/\mathbb{C}}$ identifying two copies $\mathbb{G}_a$ on $\mathrm{Spec}(\mathbb{C}[t, t^{-1}])$ but this time making $t = s^{-1}$.

Date: May 7, 2008.