$p$-ADIC ANALYTIC FAMILIES OF MODULAR FORMS

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In this course, assuming basic knowledge of algebraic number theory, elliptic modular forms, commutative algebra and topology, we will make $p$-adic study of modular forms on $GL(2)$. This is a continuation of the course-lectures of the last Winter quarter 2007. We plan to discuss the following four topics:

(1) $p$-adic analytic family of modular forms,
(2) Eichler-Shimura isomorphisms of higher weight
(3) $p$-adic measure theory via polynomial functions,

Along with these main topics, we will give a brief description of different cohomology theory we will use. In this note, all rings are supposed to have the identity, and we fix a prime $p > 2$.

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