IMAGE OF MODULAR GALOIS REPRESENTATIONS

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In this course, assuming basic knowledge of algebraic number theory, commutative algebra and topology, we study the size of the image of the modular Galois representations into $GL(2)$ attached to cusp forms. Particularly, except for the case where the representation is an induced representation of a character of a subgroup, we want to show that the image contains a principal congruence subgroup of $SL_2(A)$ for the coefficient ring A of the representation. We plan to discuss the following four topics:

(1) Lie theory of p-adic Lie groups (algebraic theory and p-adic theory),
(2) How to to show irreducibility in the p-adic case,
(3) How to measure the image via Lie theory in the p-adic case,
(4) All of the above in the Λ-adic cases.

We want to cover first three items, though we might not reach the last item within this quarter. Basic notations and terminology used in this note are explained in the lecture notes of the course given in Fall 2012 (posted in Hida's web page).

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