Please provide complete and well-written solutions to the following exercises.

Due April 24, at the beginning of class.

Assignment 4

Exercise 1. Let $a > 0$. Verify the formula
\[
\int \frac{dx}{x^2 + a} = \frac{1}{\sqrt{a}} \tan^{-1}(x/\sqrt{a}) + C.
\]

Exercise 2. Evaluate the following integrals using the method of partial fractions.
\[
\begin{align*}
\int \frac{dx}{x^2 + 2x} &= \\
\int_1^1 \frac{y + 4}{y^2 + y} dy &= \\
\int_1^{1/2} \frac{4x^2 - 21x}{(x - 3)^2(2x + 3)} dx &= 
\end{align*}
\]

Exercise 3. Compute the following integral
\[
\int \frac{xdx}{(x^2 - 1)^{3/2}}.
\]

Exercise 4. Compute the following integral:
\[
\int \ln(x^4 - 1) dx.
\]

Exercise 5. Evaluate
\[
\int_0^\infty xe^{-x} dx.
\]

Exercise 6. Compute
\[
\int_{-1}^1 \sqrt{|x|} dx.
\]

Exercise 7. Compute
\[
\int_{-1}^{2} \frac{1}{x^2} dx.
\]

Exercise 8. Compute the following integral, or show that the integral diverges.
\[
\int_0^3 \frac{dx}{(3 - x)^{3/2}}.
\]