SIX DISTRIBUTIONS

1. Binomial distribution

Use: Counting number of "successes." *Application:* n Bernoulli trials.
Range: $\{0, 1, \ldots, n\}$. Discrete. *Parameters:* n, p.
Formula: $P(S = k) = \binom{n}{k} p^k (1-p)^{n-k}$. *Mean:* np. *Variance:* $np(1-p)$.

2. Geometric distribution

Use: Waiting time to the first success. *Application:* Bernoulli trials.
Range: $\{1, 2, \ldots\}$. Discrete. *Parameter:* p.
Formulas: $P(W = k) = (1-p)^{k-1} p$ and $P(W > k) = (1-p)^k$.
Mean: $1/p$. *Variance:* $(1-p)/p^2$.

3. Poisson distribution

Uses: Counting number of "hits" in a Poisson process with rate λ, in time t. Also approximating the binomial distribution (for n large, $np < 5$).
Application: "Continuous trials" (Poisson processes).
Range: $\{0, 1, 2, \ldots\}$. Discrete. *Parameter:* $\Lambda = \lambda t$.
Formula: $P(S = k) = e^{-\Lambda} \dfrac{\Lambda^k}{k!}$. *Mean:* Λ. *Variance:* Λ.

4. Exponential distribution

Uses: Waiting time for the first "hit." Lifetime (in the absence of aging).
Application: "Continuous trials" (Poisson processes).
Range: $[0, +\infty)$. Continuous. *Parameter:* λ.
Formulas: $f(t) = \lambda e^{-\lambda t}$ and $P(X > t) = e^{-\lambda t}$. *Mean:* $1/\lambda$. *Variance:* $1/\lambda^2$.
Extension: We can modify the exponential distribution to allow for aging, if we know the hazard rate function $\lambda(t)$. $P(L > t) = e^{-\int_0^t \lambda(u)\, du}$.

5. Uniform distribution

Use: Choosing a number at random. *Application:* Interval $[a, b]$.
Range: $[a, b]$. Continuous. *Parameters:* a, b.
Formula: $f(x) = \dfrac{1}{b-a}$. *Mean:* $(a+b)/2$. *Variance:* $(b-a)^2/12$.

6. Normal distribution

Uses: Normally distributed characteristics. Approximating the binomial distribution (for n large, $np > 5$, $n(1-p) > 5$). The central limit theorem.
Applications: Many.
Range: $\mathbb{R}$. Continuous. *Parameters:* μ, σ.
Formula: $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$. *Mean:* μ. *Variance:* σ^2.

—H. B. Enderton