

## Sample Problems for the First Test

1. Say whether the statement \_\_\_\_\_ is true or false.
2. For the question \_\_\_\_\_, is the correct answer (a), (b), (c), or (d)?
3. State accurately the definition of \_\_\_\_\_.
4. (a) Show that in any field (i.e., where P1–P9 hold) with an ordering (satisfying conditions O1–O5), \_\_\_\_\_ must hold. [Pretend here that P1–P9 and O1–O5 are listed on the cover sheet.]  
  
(b) Assume that in addition, the field has the Archimedean property. Show that \_\_\_\_\_.
5. We know the triangle inequality in the form  $|x + y| \leq |x| + |y|$ . From this, show that \_\_\_\_\_ always holds.
6. For the set \_\_\_\_\_ of ordered pairs, what is the smallest number of pairs you can remove to obtain a function? To obtain a one-to-one function?
7. Assume that  $f$  is a one-to-one function from  $A$  to  $B$ . Show that \_\_\_\_\_.
8. Assume that  $A$  and  $B$  are countably infinite sets. Show that \_\_\_\_\_.
9. Show that the set \_\_\_\_\_ is uncountable.
10. Prove by induction that for every natural number  $n$ , \_\_\_\_\_.
11. Show, directly from the definition, that the sequence \_\_\_\_\_ converges to \_\_\_\_\_. That is, for an arbitrary number  $\varepsilon > 0$ , determine a natural number  $N$  such that . . . .
12. Does the sequence \_\_\_\_\_ converge to a limit, diverge to  $+\infty$ , diverge to  $-\infty$ , or none of these? Prove your answer is right.
13. Assume that  $\{a_n\}$  converges to  $K$  and  $\{b_n\}$  converges to  $L$ . Show that \_\_\_\_\_.
14. Prove or disprove that \_\_\_\_\_.

**Disclaimer:** No claim of completeness is made. The actual test need not be drawn from this list.

**Note:** The actual test cannot have fourteen separate problems. Four or five problems, each with a few parts, is more reasonable.

H. B. Enderton